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Joint Optimization of Number of Vehicles, Battery Capacity and Operations of an Electric Autonomous Mobility-on-Demand Fleet

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Executive Summary

The advent of vehicle autonomy and powertrain electrification is paving the way to the deployment of Autonomous Mobility-on-Demand (AMoD) systems whereby electric self-driving vehicles provide on-demand mobility. The operation of the electric AMoD fleet is strongly influenced by its design—e.g., smaller batteries will require to charge the vehicles more often, whilst entailing lower investment costs and energy consumption. This paper proposes to solve this tension in an integrated manner by devising a framework to jointly optimize the design and operation of an electric AMoD system, where the objective is to maximize the total profit of the fleet operator. We showcase our framework for a real-world case-study of New York City, revealing a trade-off between number of vehicles, their battery size and the amount of requests they can serve. Moreover, our results show that a significantly lower battery size can be used w.r.t. the state of the art, resulting in energy consumption reductions by up to 20%.

Keywords: smart city, mobility system, MaaS (mobility as a service)

1 Introduction

Mobility-as-a-Service (MaaS) is an emerging type of service that allows users to plan, book and pay for multiple types of mobility services through a common digital channel [1]. Companies like Uber and Lyft are gaining momentum and are becoming a concrete alternative not only to personal mobility, but also to taxi and public transport [2]. With the rise of autonomous vehicles, MaaS companies have the potential to revolutionize the transportation system through fleets of autonomous and coordinated vehicles. These Autonomous Mobility-on-Demand (AMoD) systems are envisioned to be on the streets by 2025 [3]. Moreover, as governments are pushing for the deployment of electric mobility, AMoD fleets are expected to be battery electric [4]. The operators of such systems are required to centrally control each vehicle and assign it to passenger requests, transporting the passengers from their origin to the requested destination, and rebalance the empty vehicles to match the geographical distribution of future requests, whilst recharging their batteries, as schematically shown in Fig. 1. In this context, the design of the fleet in terms of number of vehicles and individual battery size strongly influences the implementable operational strategies, as more vehicles provide more flexibility—whilst requiring larger initial investments—and larger batteries provide a longer range—but are more expensive and entail a higher energy consumption due to the heavier vehicular mass. Therefore it should be tailored to the envisioned application, which, in turn should, be chosen to make the best out of the available design. This strong coupling between design and operation calls for methods to optimize them in an integrated manner. In this paper, we propose a modeling and optimization framework that can jointly solve the optimal design and operation problem for an electric AMoD system via mixed integer linear programming.

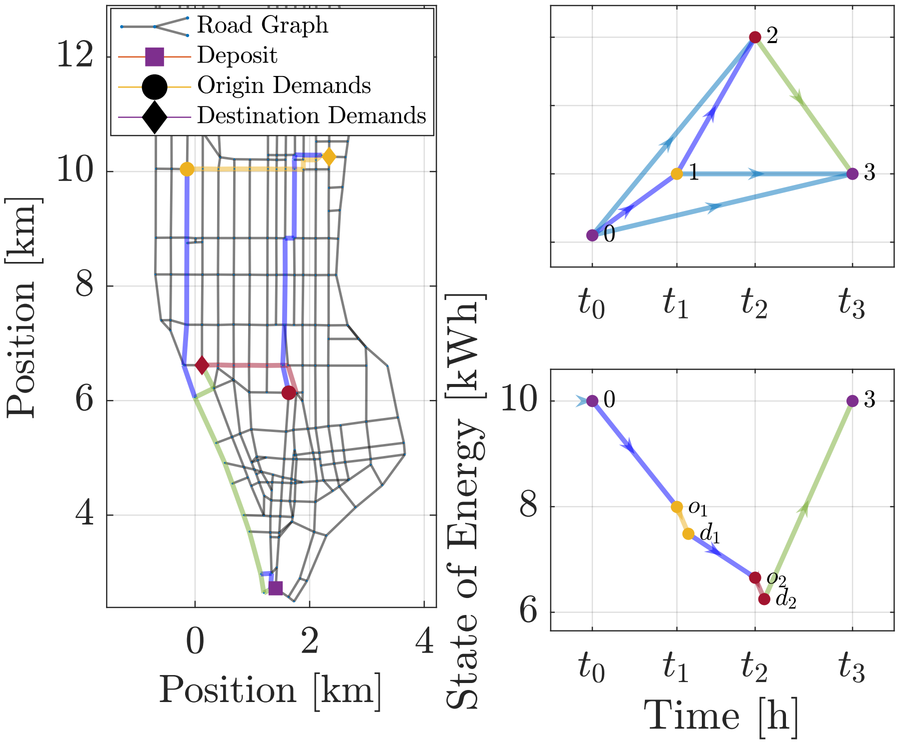

Figure 1: Single-vehicle operation on the road graph (left), directed acyclic graph representation (top-right, with non-selected possibilities in light blue), and time and battery energy plot (bottom-right). Each arc/node on the right figures represents the correspondingly colored fastest path between the nodes on the road graph. Thereby, green arcs indicate charging during a transition (in this case, inside the deposit highlighted in purple).

1.1 Literature Review

Our work contributes to two research streams, namely operation and design of AMoD systems.
To date, a significant amount of work has been devoted to studying the operation of AMoD systems [5]. For instance, real-time routing algorithms have been proposed to assign vehicles to thousands of daily requests [6]. At the same time, network flow models have been devised to optimize their operation accounting for congestion [7], and have been successfully applied to intermodal settings for different types of cars [8], or accounting for the response of private vehicles [9]. In the case of electric fleets, the operational problem must be coupled with the charge-scheduling problem. To this end, researchers have devised fast solution algorithms based on acyclic graphs [10], and accounted for the interaction with the power grid via network flow models [11, 12] and heuristic methods [13]. However, these papers do not focus on design aspects, if not via parameter studies.
The design of AMoD has also been investigated with methods ranging from Directed Acyclic Graphs (DAGs) to fluidic models [14]. Winter et al. [15] computed the amount of vehicles needed to sustain a certain quality of service through a fix start and end point model. Later, Wallar et al. [16] presented a method to optimize the fleet size in a ride-sharing framework, assuming each vehicle to have the same predefined seat-capacity. In [17], the same authors devised an improved algorithm to capture vehicles with a different seat-capacity and optimize their number and operation for ride-sharing AMoD. Turning our attention to electric AMoD, expanded network flow models inspired by [11] have been recently leveraged to optimize the charging station siting and sizing jointly with the operation of the fleet [18]. However, in all these cases the powertrain type is not considered or the battery size of the vehicle is assumed to be given, also because it allows to predefine the vehicular mass and hence the energy consumption for each arc.
In conclusion, to the best of the authors' knowledge, no optimization model exists that jointly optimizes number of vehicles, individual battery size and fleet operation, whilst guaranteeing global optimality of the solution.

1.2 Statement of Contributions

Against this background, the contribution of this paper is twofold. First, we devise an optimization model for electric AMoD systems based on acyclic graphs, where we optimize the number of vehicles and their individual battery size jointly with their operation in terms of assignment and charge scheduling, and the goal is to maximize profit. The resulting problem can be solved with global optimality guarantees via standard mixed integer linear programming algorithms [19]. Second, we showcase our framework on a real-world case-study for Manhattan, NYC, where we highlight the trade-off between number of vehicles, average battery size and requests served per vehicle.

1.3 Organization

The remainder of this paper is structured as follows: In Section 2 we first present the optimization problem formulation and our assumptions. Section 3 presents a numerical case-study for Manhattan, NYC. Finally, Section 4 concludes the paper with a brief discussion and an outlook on future research.

2 Optimization Problem Formulation

This section formulates the optimal vehicle assignment and charge scheduling problem via DAGs. Thereafter, we include variables and constraints capturing the presence of multiple vehicles and their individual battery size together with the impact on energy consumption.
We start by modeling the transportation system as a directed graph $\mathcal{G}' = (\mathcal{V}', \mathcal{A}')$, where the set of arcs $\mathcal{A}'$ represents road links, whilst the set of vertices $\mathcal{V}'$ contains intersections. We also indicate D_{mn} and T_{mn} as the distance and travel time, respectively, of road segments between road intersections $m, n \in \mathcal{V}'$. We denote a set of travel requests by $\mathcal{I}$ with $i \in \mathcal{I} := \{1, 2, ..., I\}$ the set of transportation requests. In order to model the demanded trips, let the triple $(o_i, d_i, t_i^{\mathrm{start}})$ denote a requested trip, where t_i^{start} is the requested pick-up time, whilst $o_i, d_i \in \mathcal{V}'$ are the origin and destination nodes of request i, respectively. In the area under consideration there are C charging stations, whereby each station $c \in \mathcal{C} := \{1, 2..., C\}$ is located at vertex $n_c \in \mathcal{V}'$. For each arc in $\mathcal{A}'$ we assume the driving pattern, and hence also the travel time, to be fixed and known in advance for each time of the day. Finally, we assume that vehicles drive through the fastest path when traveling from one location to another. Yet other criteria can be readily implemented to predefine the paths between locations.
In order to study the fleet design and operation problem in a mathematically tractable fashion, we construct an acyclic directed graph $\mathcal{G}$, similar to [20], containing deposits and transportation requests. To include deposits, we define a new set of requests $\mathcal{I}^+ := \{0, 1, 2..., I, I+1\}$ where the deposits are the first and last requests that have to be served so that vehicles start and conclude their schedules in a deposit. Specifically, graph $\mathcal{G}'$ (left) describes the geography of the road network, which is transformed in $\mathcal{G}$, a DAG (top-right). The bottom-left figure represents the computed solution of the sequence in which the requests are served. For example, in Fig. 1, the arcs in $\mathcal{G}'$ connecting the destination of request 1 d_1, and the origin of request 2 o_2, represent a path, which is captured by a single arc in $\mathcal{G}$. This way we can capture the transitions between requests: Each arc $(i, j) \in \mathcal{V}$ represents the transition from the destination of i to the origin of j, and it is characterized by the travel time and distance t_{ij}^{fp} and d_{ij}^{fp}, respectively, on the fastest path. If $i = j$, then t_{ii}^{fp} is merely the fastest time to serve i. Finally, given the set of $K \in \mathbb{N}$ vehicles $\mathcal{K} := \{1, 2..., K\}$, to capture whether vehicle $k \in \mathcal{K}$ serves request i and then request j, we set the binary tensor $X_{ij}^k = 1$ and to 0 otherwise. Furthermore, if between the two requests vehicle k charges its battery at charging station c, we set the binary tensor $S_{ijc}^k = 1$ and 0 otherwise, whilst quantifying the amount of battery charged with the non-negative-valued tensor C_{ijc}^k. The example in Section 2.1 below illustrates the solution for the simple case shown in Fig. 1. We refer the reader to the Appendix for a more detailed description of each optimization variable.

2.1 Example

In the example shown in Fig. 1, serving first request 1 and then request 2 with vehicle 1 through the fastest path is expressed by $X_{12}^1 = 1$. It means that the vehicle will transition between the two requests and then idle for the rest of the time. During the last transition between request 2 and 3 (the deposit), where $X_{23}^1 = 1$, we also charge the vehicle at charging station 1 in the deposit. $S_{231}^1 = 1$, depicted in green, indicating that during the transition, charging occurs, and $C_{231}^1 = 3.7\,\mathrm{kWh}$ equal to the amount of energy recharged by vehicle 1 in charging station 1.

2.2 Objective Function

In this paper, we set as optimization objective to maximize profit: We minimize the total cost of ownership of the fleet minus the revenues by expressing the total cost of ownership as a linear combination of initial and operational costs. Formally, we define the objective function as follows:

$$J = \sum_{k\in\mathcal{K}} \left(p_0^k + p_{\text{el}} \cdot \sum_{i,j\in\mathcal{I}^+} \sum_{c\in\mathcal{C}} C_{ijc}^k \right) - \sum_{i\in\mathcal{I}^+} b_{\text{r}}^i \cdot p_i, \tag{1}$$

where p_0^k is the amortized cost to purchase vehicle k, p_{el} is the price of electricity, C_{ijc}^k is the energy charged by vehicle k at charging station c during transition i-j, b_{r}^i is a binary variable indicating if request i was served or not, and p_i the revenue realized by serving the i-th request. We model the revenue generated by each request as an affine function with respect to time and distance required to serve it:

$$p_i = \alpha + \beta \cdot d_{ii}^{\text{fp}} + \gamma \cdot t_{ii}^{\text{fp}} \quad \forall i \in \mathcal{I}, \tag{2}$$

with α equal to the base fare, β to the cost per unit distance and γ to the cost per unit time.
We then define vehicle k's amortized cost (the process of gradually writing off the initial costs) as an affine function of its battery energy capacity E_{b}^k, amortized on the total vehicle lifetime τ_{v}:

$$p_0^k = \frac{p_{\text{v}} \cdot b_{\text{v}}^k + p_{\text{b}} \cdot E_{\text{b}}^k}{\tau_{\text{v}}} \quad \forall k \in \mathcal{K}, \tag{3}$$

where p_{b} is the price per unit energy of battery, b_{v}^k is a binary variable equal to 1 if vehicle k is used, p_{v} is the price to purchase the whole vehicle excluding the battery. Hereby, the ratio $p_{\text{v}}/\tau_{\text{v}}$ captures the daily fixed cost per vehicle that might include factors like insurance, storage and maintenance.

2.3 Transition Constraints

We allow vehicle k to charge an amount C_{ijc}^k during transition i-j, only if it performs the transition and visits station c:

$$\sum_{c\in\mathcal{C}} S_{ijc}^k \leq X_{ij}^k \quad \forall i,j \in \mathcal{I}^+,\ \forall k \in \mathcal{K} \tag{4}$$

$$C_{ijc}^k \leq M \cdot S_{ijc}^k \quad \forall i,j \in \mathcal{I}^+,\ \forall c \in \mathcal{C},\ \forall k \in \mathcal{K}, \tag{5}$$

where M is a sufficiently large number, in line with the Big M formulation [21]. To respectively initialize and shut down the vehicles in the deposit, we define the two parameters f and l, so that $f_j^k, l_j^k = 0\ \forall i,j \in \mathcal{I}$ and $f_0^k = l_{I+1}^k = 1\ \forall k \in \mathcal{K}$. Each request cannot be served more than once. In particular, demand j can only be served after a single previous demand i has been served,

$$\sum_{i\in\mathcal{I}^+, k\in\mathcal{K}} X_{ij}^k + \sum_{k\in\mathcal{K}} f_j^k \leq 1 \quad \forall j \in \mathcal{I}^+. \tag{6}$$

At the same time, after request i, only a single request j can be served,

$$\sum_{j\in\mathcal{I}^+, k\in\mathcal{K}} X_{ij}^k + \sum_{k\in\mathcal{K}} l_i^k \leq 1 \quad \forall i \in \mathcal{I}^+. \tag{7}$$

Finally, we ensure that the vehicle dropping off request j starts the next transition from the same request j with

$$\sum_{i\in\mathcal{I}^+} X_{ij}^k - \sum_{l\in\mathcal{I}^+} X_{jl}^k = f_j^k + l_j^k \quad \forall j \in \mathcal{I}^+,\ \forall k \in \mathcal{K}. \tag{8}$$

We note that in the previous equations (6)–(8) the terms $f_j^k$and l_j^k are always null for $i, j \in \mathcal{I}$, but equal to 1 if $i, j \in \{0, I+1\}$, i.e., at the beginning or end of the schedule. In these cases, the initial or final transition is initialized or finalized in a deposit through either one of the two parameters.

2.4 Energy Constraints

We derive and describe the constraints related to the energy consumption of the vehicles as follows: We set the state of energy of vehicle k at the end of trip j e_j^k as its state of energy at the end of the previous trip i minus the energy required to transition from i to j and serve j E_{ij}^k, and plus the energy charged at station $c \in \mathcal{C}$ as

$$e_j^k = e_i^k - E_{ij}^k + \sum_{c\in\mathcal{C}} C_{ijc}^k \qquad \forall i,j \in \mathcal{I}^+, \forall k \in \mathcal{K} | X_{ij}^k = 1. \tag{9}$$

We use the big M formulation to re-write (9) as

$$e_j^k \geq e_i^k - E_{ij}^k + \sum_{c\in\mathcal{C}} C_{ijc}^k - M \cdot (1 - X_{ij}^k) \qquad \forall i,j \in \mathcal{I}^+, \forall k \in \mathcal{K} \tag{10}$$

$$e_j^k \leq e_i^k - E_{ij}^k + \sum_{c\in\mathcal{C}} C_{ijc}^k + M \cdot (1 - X_{ij}^k) \qquad \forall i,j \in \mathcal{I}^+, \forall k \in \mathcal{K}. \tag{11}$$

We define the energy to perform transition i-j as

$$E_{ij}^k = \begin{cases} D_{ij}^{\mathrm{fp}} \cdot \Delta e^k \;\; \forall i,j \in \mathcal{I}^+, \forall k \in \mathcal{K} | \sum_{c\in\mathcal{C}} S_{ijc}^k = 0 \\ (D_{ij}^{\mathrm{fp}} + \Delta D_{ijc}^{\mathrm{go2S}}) \cdot \Delta e^k \\ \qquad \forall i,j \in \mathcal{I}^+, \forall c \in \mathcal{C}, \forall k \in \mathcal{K} | S_{ijc}^k = 1, \end{cases} \tag{12}$$

where Δe^k is the consumption per unit distance of vehicle k, D_{ij}^{fp} the fastest path distance between the location of the drop-off of i and the pick-up location on j, $\Delta D_{ijc}^{\mathrm{go2S}}$ is the additional distance traveled to pass by charging station c. We reformulate (12) with the big M formulation as follows:

$$E_{ij}^k \geq \Delta e^k \cdot D_{ij}^{\mathrm{fp}} \;\; \forall i,j \in \mathcal{I}^+, \forall k \in \mathcal{K} \tag{13}$$

$$E_{ij}^k \leq \Delta e^k \cdot D_{ij}^{\mathrm{fp}} + M \cdot \sum_{c\in\mathcal{C}} S_{ijc}^k \qquad \forall i,j \in \mathcal{I}^+, \forall k \in \mathcal{K} \tag{14}$$

$$E_{ij}^k \geq \Delta e^k \cdot (D_{ij}^{\mathrm{fp}} + \Delta D_{ijc}^{\mathrm{go2S}}) - M \cdot (1 - S_{ijc}^k) \qquad \forall i,j \in \mathcal{I}^+, \forall k \in \mathcal{K}, \forall c \in \mathcal{C} \tag{15}$$

$$E_{ij}^k \leq \Delta e^k \cdot (D_{ij}^{\mathrm{fp}} + \Delta D_{ijc}^{\mathrm{go2S}}) + M \cdot (1 - S_{ijc}^k) \qquad \forall i,j \in \mathcal{I}^+, \forall k \in \mathcal{K}, \forall c \in \mathcal{C}, \tag{16}$$

where (13) always applies, (14) becomes inactive if $\sum_c S_{ijc}^k = 0$, whilst (15) and (16) become active if $\sum_c S_{ijc}^k = 1$. Starting from the mechanical energy consumption and assuming a fairly constant battery-to-wheels efficiency, we can formulate the vehicle's consumption per unit distance as an affine function of its mass [22, Ch. 2]. Since mass is in turn an affine function of the battery size, the vehicle consumption per unit distance Δe^k is

$$\Delta e^k = \Delta e_0 + \Delta e_{\mathrm{b}} \cdot E_{\mathrm{b}}^k \quad \forall k \in \mathcal{K}, \tag{17}$$

with Δe_0 being the base vehicle consumption and Δe_b a linear term. Thereafter, the battery size of each vehicle must be larger than the energy stored at any point in time:

$$E_\mathrm{b}^k \geq e_j^k \qquad \forall j \in \mathcal{I}^+,\ \forall k \in \mathcal{K} \tag{18}$$

$$e_j^k \geq 0 \qquad \forall j \in \mathcal{I}^+,\ \forall k \in \mathcal{K}. \tag{19}$$

Finally, we enforce an energy balance for each vehicle at the beginning and end of the schedule,

$$e_0^k f_0^k = e_{I+1}^k l_{I+1}^k \ \ \forall k \in \mathcal{K}. \tag{20}$$

2.5 Time Constraints

To determine which transitions are possible, we define T_{ijc}^k as the time required by vehicle k to complete the transition i-j passing through charging station c, and recharging C_{ijc}^k amount of energy as

$$T_{ijc}^k = t_{ij}^\mathrm{fp} + \Delta T_{ijc}^\mathrm{go2S} \cdot S_{ijc}^k + \frac{C_{ijc}^k}{P_\mathrm{ch}}, \tag{21}$$

where the first term t_{ij}^fp represents the time to complete transition i-j, the second term $\Delta T_{ijc}^\mathrm{go2S} \cdot S_{ijc}^k$ captures the additional time required in case charging occurs during the transition (going to charging station c after serving i and before picking-up j), and the last term is the time required to charge the amount C_{ijc}^k, with P_ch being the constant charging power.
We now impose time constraints to allow only feasible transitions. Since travel, deviation and charging times can be efficiently pre-computed via standard shortest path algorithms, we can pre-compute which transitions are feasible and directly eliminate such variables. Specifically, for each transition i-j and vehicle k we have

$$X_{ij}^k \leq \begin{cases} 1 & \text{if } t_{ij}^\mathrm{fp} \leq t_{ij}^\mathrm{ava} \\ 0 & \text{otherwise} \end{cases} \quad \forall i,j \in \mathcal{I}^+,\ \ \forall k \in \mathcal{K}, \tag{22}$$

where $t_{ij}^\mathrm{ava} := t_j^\mathrm{start} - t_i^\mathrm{end}$ is the available time between the drop-off of customer i and pick-up time of customer j. Via this upper bound, we obtain a DAG with triangular adjacency matrix as described in [10, 20]. Similarly, we allow a deviation to charging station c within transition i-j only if there is enough time available:

$$S_{ijc}^k \leq \begin{cases} 1 & \text{if } t_{ij}^\mathrm{fp} + \Delta T_{ijc}^\mathrm{go2S} \leq t_{ij}^\mathrm{ava} \\ 0 & \text{otherwise} \end{cases} \quad \forall i,j \in \mathcal{I}^+,\ \ \forall k \in \mathcal{K}. \tag{23}$$

Finally, we bound the amount of energy that can be charged when deviating to station c as

$$C_{ijc}^k \leq \begin{cases} \hat{C}_{ijc}^k & \text{if } t_{ij}^\mathrm{fp} + \Delta T_{ijc}^\mathrm{go2S} \leq t_{ij}^\mathrm{ava} \\ 0 & \text{otherwise} \end{cases} \quad \forall i,j \in \mathcal{I}^+,\ \ \forall k \in \mathcal{K}, \tag{24}$$

where $\hat{C}_{ijc}^k = (t_{ij}^\mathrm{ava} - t_{ij}^\mathrm{fp} - \Delta T_{ijc}^\mathrm{go2S}) \cdot P_\mathrm{ch}$ is the maximum amount of energy that can potentially be charged during transition i-j at charging station c, if all the available time left were used to charge.

2.6 Number of Vehicles

This section introduces constraints to capture which vehicles are used and which can be discarded from the fleet. First, we set the binary variable $b_\mathrm{v}^k = 1$ to indicate whether vehicle k is being used, or to 0 if the vehicle stays in the deposit and is hence not needed. As previously explained in Section 2 above, deposits are modeled as the first and the last requests to be served and included in $\mathcal{I}^+$. We determine whether a vehicle is being used by

$$\sum_{i,j \in \mathcal{I}^+} X_{ij}^k \leq 1 - M \cdot b_\mathrm{v}^k \quad \forall k \in \mathcal{K}, \tag{25}$$

whereby, if the vehicle is not being used, it will transition only once from the deposit to the deposit without using energy. Finally, to avoid multiple solutions, we use vehicle k before using vehicle $k+1$:

$$b_\mathrm{v}^{k+1} \leq b_\mathrm{v}^k \quad \forall k \in \mathcal{K}. \tag{26}$$

Given the objective (1), if a vehicle is not used, its battery size will be automatically set to 0.

2.7 Problem Formulation

Summarizing, we formulate the maximum-profit design and operation problem for an electric AMoD fleet as follows:

Problem 1 (Joint Design and Operation Optimization)**.** *Given a set of transportation requests $\mathcal{I}$, the number of vehicles, their battery size, and their operations maximizing the total profit result from*

$$\min J$$
$$\text{s.t. } (1)-(8),\ (10)-(11),\ (13)-(26).$$

Problem 1 is a mixed integer linear program that can be solved with global optimality guarantees by off-the-shelf optimization algorithms.

2.8 Discussion

A few comments are in order. First, we consider travel times on the road digraph $\mathcal{G}'$ to be given. This assumption is in order for a small fleet as the one under consideration, whose routing strategies do not significantly impact travel time and hence exogenous traffic. However, we can still capture varying levels of exogenous traffic during the course of the day by simply looking at time-dependent traffic data. Second, we assume that charging stations always have a free spot, which is acceptable if we consider the plugs to be owned by the operator. We leave the inclusion of constraints to avoid potentially conflicting charging activities by multiple vehicles to future research. Third, considering design aspects, optimizing the fleet for a specific scenario may render its design not feasible for another one. This problem can be addressed by either a robust optimization or by solving the problem for multiple scenarios and picking the most conservative one. Finally, from a computational perspective, Problem 1 is NP-hard, therefore its solution can take a significant amount of time, as shown in the results Section 3 below. As the focus of this study is on framing the design and operation problem in a joint fashion, we will leverage off-the-shelf algorithms to compute globally optimal solutions (or at least within a known optimality gap), whilst leaving the development of ad-hoc heuristic solution methods that are computationally more efficient to future research.

3 Results

This section showcases our framework for Manhattan, NYC. First, in Section 3.1, we study a small fleet and demand set (5 vehicles for 60 requests in 24 hours) that allows us to jointly visualize the design and operation solution. Thereafter, to tackle a larger problem instance in Section 3.2, inspired by the central limit theorem, we find a solution for multiple scenarios containing sampled demands to obtain a discretized distribution of the optimal design and performance. Specifically, we use the road network shown in Fig. 2, consisting of 357 nodes and 1006 links, which was constructed using a version based on OpenStreetMaps [23].
The travel requests were supplied by technology providers authorized under the Taxicab & Livery Passenger Enhancement Programs to the NYC Taxi and Limousine Commission. The data set is built using historical data of taxi rides that occurred in March 2018. The values of all the parameters used are in Table 1 together with their source. Finally, we assume that the AMoD operator has a privately owned charging infrastructure, evenly distributed in the area of interest as shown in Fig. 2. To parse and solve Problem 1, we respectively use Yalmip [30] and Gurobi 8.1 on a Laptop with 16 GB of RAM and an Intel i7-9750H processor. All the solutions shown were obtained within a global optimality gap of less than to 1%.

3.1 Small Scenario

In this small scenario, we extract 60 random travel requests from the data set, and we initialize a maximum of 5 vehicles in the deposit with an initial energy of 10 kWh each. We solve Problem 1 twice, the first one with a cheap base price per car of $p_\mathrm{v} = 15\,\mathrm{kEur}$, and the second one with an expensive base price of $p_\mathrm{v} = 30\,\mathrm{kEur}$, both amortized over a period of 3 years. Fig. 3 and Fig. 4 show the operations in the two scenarios. Note that to ease the readability, in both figures we omit the origin nodes and directly connect the destinations of two nodes. Moreover, the green color indicates charging during the transition, whilst other colors represent one of the vehicle. Comparing the two figures, we see that if vehicles are cheap, the solution converges to three vehicles with a rather small battery. In contrast, when the base price is more expensive, we obtain only two vehicles with a larger battery size as they need to serve more requests. The optimal battery size corresponds to the highest state of energy point reached by each vehicle in the graph, which is approximately 14, 12 and 12 kWh for cheap base vehicles and 19 and17 kWh for more expensive vehicles. Hereby, we see that a vehicle with battery size of 14 kWh consumes 7% less per unit distance compared to a 19 kWh vehicle, highlighting the impact of battery sizing on energy efficiency.

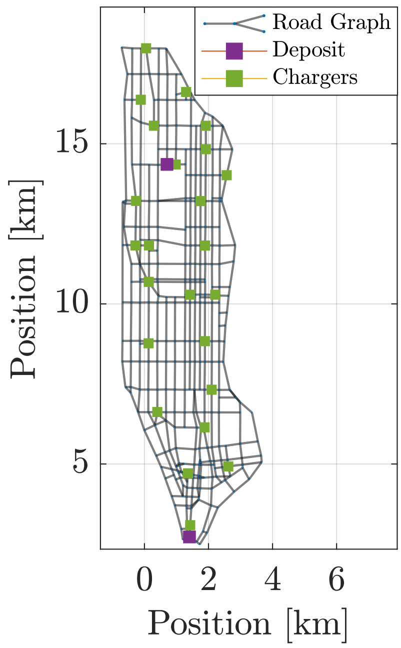

Figure 2: NYC road network. The purple nodes are deposits, the green nodes are charging stations.

Parameter	Value	Unit	Reference
$\tau_{\rm v}$	1095	days	[24]
$p_{\rm v}$	$21,000$	€	[25]
$p_{\rm batt}$	200	€/kWh	[26]
$p_{\rm el}$	0.207	€/kWh	[27]
$P_{\rm ch}$	6	kWh/h	[28]
Δe_0	0.0125	kWh/km	[22]
$\Delta e_{\rm b}$	0.003	1/km	[22]
α	2.55	€/ride	[29]
β	1.5	€/km	[29]
γ	0.35	€/min	[29]

Table 1: Values of Parameters

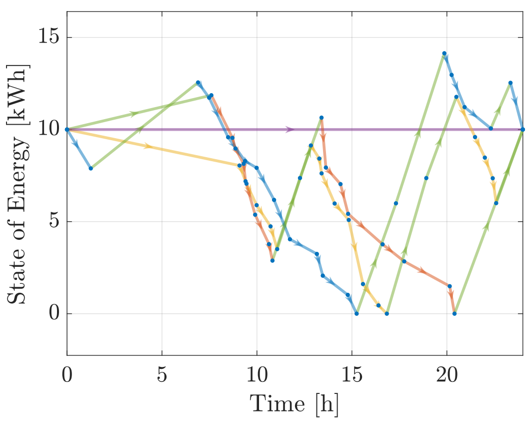

Figure 3: Design and operation of a fleet of vehicles with $p_{\rm v} = 15\,\text{kEur}$. The optimal battery size of the three vehicles used is 14, 12 and 12 kWh.

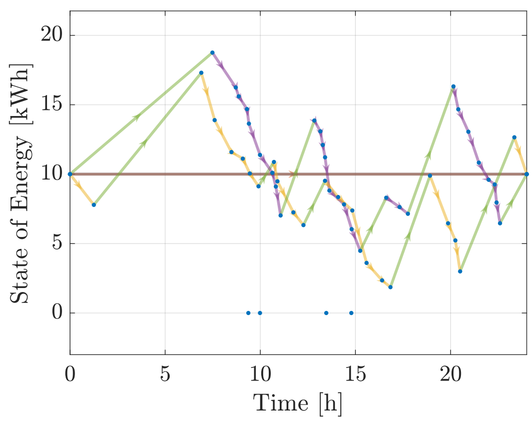

Figure 4: Design and operation of a fleet of vehicles with $p_{\rm v} = 30\,\text{kEur}$. The optimal battery size of the two vehicles used is 19 and 17 kWh. In this case, four demands are dropped, as indicated by the four disconnected points.

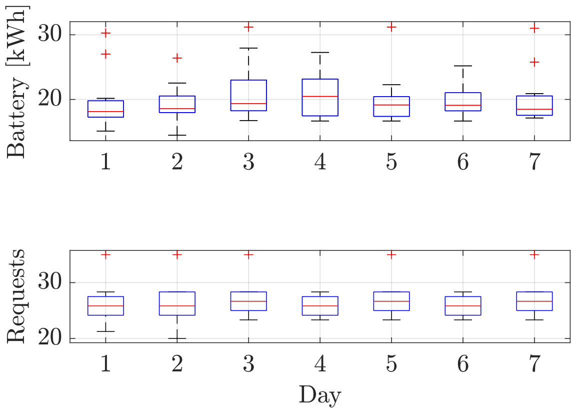

Figure 5: Optimal solution of Problem 1. On the top there is the distribution of the average battery size of the fleet. Below it, there are the average number of requests served per vehicle.

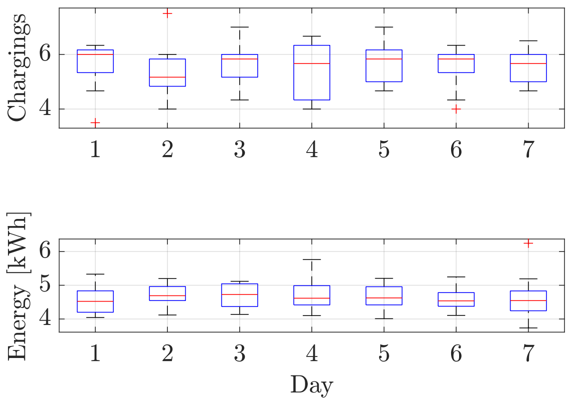

Figure 6: Optimal solution of Problem 1. On the top there is the average number of daily detours of a vehicle to a charging station. Below it, there is the average amount of energy charged during a detour.

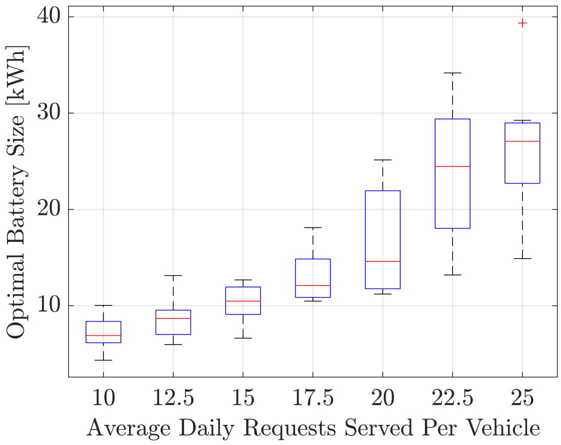

Figure 7: Optimized battery size w.r.t. average number of daily requests served per vehicle. When fewer requests are served, the required battery size decreases and the energy consumption per unit distance diminishes accordingly.

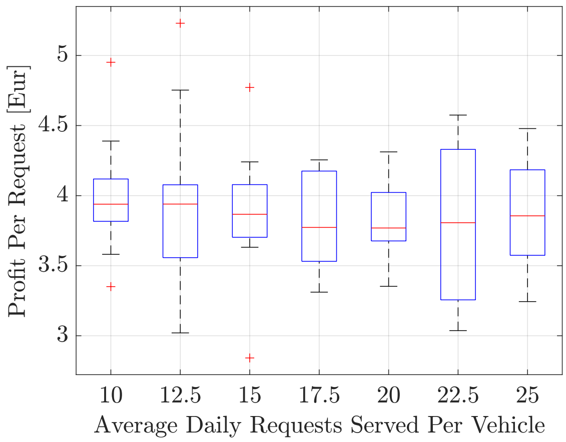

Figure 8: Profit generated per vehicle (revenues minus costs) w.r.t. the number of daily requests served per day.

3.2 Case Study of Manhattan

In this section, we randomly extract 15 subsets of 180 requests from the data set for each day between March 1 and 7, 2018. We then solve each instance of Problem 1 to obtain a discretized distribution of the solution. Solving each instance took a computational time of approximately 2 h.

Fig. 5 and Fig. 6 show the optimal solution of Problem 1, where we normalize the number of vehicles with the number of requests. On average, each car serves approximately 25 requests per day. The optimal battery size distribution lies within 20-25 kWh, reasonably smaller compared to commercial vehicles. This feature comes at a cost: Each vehicle charges a small amount of energy multiple times a day.

Finally, to investigate the trade-off between number of vehicles, battery size and profit, we solve Problem 1 fixing the number of active vehicles used, hence removing the influence of the price per vehicle p_{v} on the solution. Hereby, we only use data from March 3, 2018. Fig. 7 shows the correlation between number of requests served and optimal battery size, indicating that a vehicle needs a smaller battery size if it has to serve fewer requests per day. Conversely, the battery size is a free variable and is consequentially scaled by the algorithm so that the cost of operation is minimized. We then proceed to investigate the impact on profitability. To draw a fair comparison, we plot the profit (revenues minus costs) per car per request served when the base price is $p_{\mathrm{v}} = 21\,\mathrm{kEur}$: Fig. 8 shows how profit tends to be constant for different fleet sizes. On the one hand, by increasing the number of daily requests served, the revenues increase. On the other hand, not only the traveled distance increases, but also the energy consumption per unit distance, counteracting the benefits of serving more requests. This equilibrium where changing the number of vehicles would not change the profitability of the system results from the base price of $p_{\mathrm{v}} = 21\,\mathrm{kEur}$, which can be intended as a threshold whereby lower base prices would favor larger fleets with a small battery size, whilst higher base prices would result in smaller fleets with a larger battery size.

4 Conclusions

In this paper, we presented a framework to jointly optimize the number of vehicles, their battery size and the operation of an electric autonomous mobility-on-demand fleet. In particular, we coupled the operation problem of the fleet in terms of vehicle assignment and charge scheduling with the design problem entailing the number of vehicles and their individual battery size. All in all, we showed that these two problems can be jointly solved via mixed integer linear programming with global optimality guarantees. We showcased our framework in a real-world case-study for Manhattan, NYC. To cope with the NP-hard problem structure and the resulting computational complexity, inspired by the central limit theorem, we solved smaller problem instances for sampled demand subsets and gathered statistical information on the optimal fleet design. Our results revealed that i) A fleet with average battery size of 20-25 kWh is sufficient and, compared to a fleet of vehicles with a battery size of 40-50 kWh, can lead to a decrease in energy consumption of up to 20%. ii) There exists a threshold base vehicle price under which the number of vehicles can be increased without sacrificing the overall profit of the operator.
This paper opens the field to the following extensions: First, the acyclic digraph framework makes the inclusion of ride-sharing a natural next step. Second, we would like to account for intermodal settings, whereby transportation requests are serviced jointly with public transit. Finally, we would like to devise ad-hoc solution algorithms inspired by [16] to allow a more efficient solution of the optimization problem.

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Appendix

We define the optimization variables that we use to formulate Problem 1 as follows:

- $X \in \{0,1\}^{I\times I\times K}$: a transition tensor with binary variable X_{ij}^k equal to 1 if transition between request i and request j occurs and is served by vehicle k, 0 otherwise.
- $S \in \{0,1\}^{I\times I\times C\times K}$: a charging station tensor with binary variable S_{ijc}^k equal to 1 if charging occurs at charging station c, during the transition $i-j$, served by vehicle k, 0 otherwise.
- $C \in \mathbb{R}+^{I\times I\times C\times K}$: a recharge tensor with continuous variable C_{ijc}^k equal to the energy charged by vehicle k between transition i and j at charging station c,
- $E \in \mathbb{R}+^{I\times I\times K}$: an energy transition tensor with continuous variable E_{ij}^k equal to the energy spent to go from i to j and serve j, expressed in kWh.
- $e \in \mathbb{R}+^{I\times K}$: an energy vector with element e_i^k equal to the energy stored by vehicle k after serving demand i, expressed in kWh.
- $E_{\mathrm{b}} \in \mathbb{R}+^{K}$: a battery vector with E_{b}^k representing the maximum energy stored by vehicle k during the simulation (battery of vehicle k), expressed in kWh.
- $b_{\mathrm{v}} \in \{0,1\}^K$: an active vehicles vector with binary variable b_{v}^k equal to 1 if vehicle k is active (used), 0 otherwise.
- $b_{\mathrm{r}} \in \{0,1\}^I$: a served requests vector with binary variable b_{r}^i equal to 1 if request i is served, 0 otherwise.

- $f \in \{0,1\}^{I \times K}$: a parameter used to initialize vehicle k in the deposit, with f_j^k binary element equal to 1 if request j is the starting deposit, 0 otherwise. In other words, $f_0^k = 1$, $f_j^k = 0$ with $j \in \mathcal{I}^+ \backslash \{0\}$
- $l \in \{0,1\}^{I \times K}$: a parameter used to finalize vehicle k in the deposit, with f_j^k binary element equal to 1 if request j is the ending deposit, 0 otherwise.In other words, $l_{I+1}^k = 1$, $f_j^k = 0$ with $j \in \mathcal{I}^+ \backslash \{I+1\}$

In case of free floating fleets without deposits, f and l can be considered as optimization variables. If deposits are present, as in the present paper, they are used to initialize and finalize the vehicles in the corresponding deposit.

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