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Vertical Vibration Analysis on Electric Vehicle with Suspended In-wheel Motor Drives

Wei TONG, *Zhichao HOU**

(Corresponding author) houzc@mail.tsinghua.edu.cn , State Key Laboratory of Automotive Safety and Energy, Tsinghua University, Beijing,China,100084

Abstract

In-wheel motor drive (IWMD) is currently a hot topic in electric vehicle research and development for its remarkable advantages. But the extra wheel mass from the motor worsens vehicle ride comfort and road holding. Motor vibration is another problem, which does harm to motor performance and service life.

As a trial response to the above problems, a 1/4 car model is established for electrical vehicles driven by suspended in-wheel motors. Five indexes are then employed or defined to evaluate vehicle performances and motor vibration, which are the road holding, vehicle suspension deflection, motor suspension deflection and vertical vibration accelerations of the body and the motors. After sensitivity analysis on motor suspension parameters, optimization is then performed with two different cost functions defined in terms of the amplitude-frequency characteristic of each of the indexes. One function, noted as A-Opt, is the area enclosed by a characteristic curve and frequency axis. The other function, noted as P-Opt, is the summation of the peak values of the characteristic curve. Both the transfer function and the variance of the response are used to describe the vehicle performance. With the former function, focus is mainly put on the frequency response characteristics of the vehicle. With the latter one on the other hand, attention is paid to average amplitude variation for the vehicle on a typical road. Results are compared with those of equivalent electrical vehicles driven either by a centre motor or by fixed in-wheel motors.

Results demonstrate that a properly designed motor suspension can effectively improve the performances of vehicles with in-wheel motor drive, including ride comfort, road holding and motor working condition. Furthermore, the second optimization strategy has advantages over the first one with regard to vibration reduction and resonance elimination.

Keywords: battery electric vehicle, motor design, optimization, vehicle performance, simulation

1 Introduction

Recently, the in-wheel motor (IWM) drive for vehicles has been a hot topic in electric vehicle research and development due to many remarkable advantages, such as quick motor responses, precise torque generation, high transmission efficiency and easy implementation of ABS, TCS and ESC [1].

However, the extra wheel mass deteriorates vehicle performances in ride comfort, road holding and motor working conditions. There are very limited studies on these issues, although numerous researches have been conducted on design and control of IWM for electrical vehicles.

A review introduced some possible solutions to reduce the impact of extra wheel mass to the vertical performances of vehicles with IWMs [2]. A linear quadratic optimal control was applied for an active suspension to compress the negative influences of the extra wheel mass on vehicle vertical vibration [3]. Various vibration absorbing mechanisms were also applied to improve the performances of tire road holding and vehicle ride comfort, including a conventional dynamic vibration absorber fixed on the steering knuckle[4], the one with the stator of the IWM suspended between the axle and the vehicle body [5], and the Bridgestone's dynamic damper type in-wheel motor system (BDIMS) [6,7]. In the BDIMS, the IWM was suspended by a parallel spring-damper suspension. As a result, the motor serves as not only a power source, but also a tuned mass damper (TMD). These researches focused on potential improvement on vehicle performance by introducing motor suspension. But they usually neglected other impacts from the motor suspension to vehicle behaviour, such as the resonance peak introduced by motor suspension, and did not pay any attention to the vibration of the in-wheel motor. It is thus desirable to optimize the motor suspension based on a compromise between vehicle performance and motor vibration.

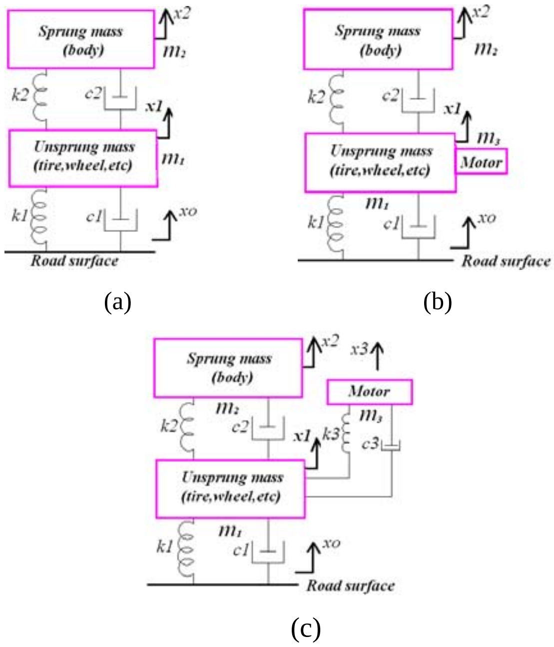
For a vehicle with four suspended IWMs, this paper addresses the impacts of motor suspension on road holding, ride comfort and motor vibration. After sensitivity analysis on motor suspension parameters, optimizations with two different strategies are performed in terms of several performance indexes. The results are compared with those of the vehicles driven by traditionally fixed IWMs or by a central motor, to show the improvement due to motor suspension.

2 Equations and performance indexes

In this section, we firstly introduce several quarter- car models for vehicles respectively driven by a central motor, four fixed IWMs or four suspended IWMs. The equations on vertical vibration of the vehicles are then derived. Five indexes are employed or defined to evaluate the performances of the vehicles and the motors.

2.1 Car models

Quarter-car models are employed to describe the vertical vibration of vehicles driven by a central motor, four fixed IWMs or four suspended IWMs, as shown in Fig.1.



(a) Vehicle with a central motor drive (CMD),
(b) Vehicle with fixed IWM drives (FIWMD),
(c) Vehicle with suspended IWM drives (SIWMD)

Fig.1 Quarter-car models

In the figure, m_1 , m_2 and m_3 represent the unsprung mass, the sprung mass and the mass of each IWM assembly, respectively. An IWM assembly might include a planetary gear besides a motor. k_1 , k_2 and k_3 separately represent the spring rates of the tire, the vehicle suspension and the motor suspension, and c_1 , c_2 and c_3 are the related damping coefficients. F_s and x_0 denote the vertical force acted on the vehicle body and road profile excitation on the tire, while x_1 , x_2 and x_3 are separately the vertical displacements of the wheel, the body and the motor.

The dynamic equations of the models can be derived as follows. For the vehicle with a central motor drive,

$$m_2\ddot{x}_2 = F_s - u_s \tag{1}$$

$$m_1\ddot{x}_1 = u_s - F_r \tag{2}$$

Here F_s , u_s and F_r are forces applied to the vehicle body, between the vehicle body and wheel, and to the tire, respectively.

For a vehicle driven by four fixed in-wheel motors, one has with the quarter-car model

$$m_2\ddot{x}_2 = F_s - u_s \tag{3}$$

$$(m_1 + m_3)\ddot{x}_1 = u_s - F_r \tag{4}$$

For a vehicle driven by suspended in-wheel motors, one has

$$m_3\ddot{x}_3 = -u_m \tag{5}$$

$$m_2\ddot{x}_2 = F_s - u_s \tag{6}$$

$$m_1\ddot{x}_1 = u_s + u_m - F_r \tag{7}$$

In the equation, u_m is the suspension force between the in-wheel motor and the wheel. Other symbols are the same as defined with Eqs. (1) and (2).

If there is no external force acted on the vehicle body, $F_s = 0$. According to the definition of each variable, one has in the Laplace domain,

$$\hat{u}_s = \left(c_2 s + k_2\right)\cdot\left(\hat{x}_2 - \hat{x}_1\right) \tag{8}$$

$$\hat{u}_m = \left(c_3 s + k_3\right)\cdot\left(\hat{x}_3 - \hat{x}_1\right) \tag{9}$$

$$\hat{F}_r = \left(c_1 s + k_1\right)\cdot\left(\hat{x}_1 - \hat{x}_0\right) \tag{10}$$

Here symbol '^' represents the Laplace transform to the variable under it, the parameter *s* is a complex number.

2.2 Performance indexes

Five indexes are adopted to evaluate vehicle performances and working condition of the in-wheel motor. The indexes are related to the dynamic tire contact force, vehicle suspension deflection, vehicle body vertical vibration acceleration, motor suspension deflection and motor vertical vibration acceleration. A concept of motor suspension deflection is introduced to reflect space requirement on the wheel to hold the motor and other related components. Motor vibration acceleration is chosen to describe working condition of the in-wheel motor.

Each of the index is defined in frequency domain by certain transfer function, to reveal its frequency response characteristics, especially the resonance peaks related to vehicle vibratory performances. To understand average amplitude variation of the indexes as the vehicle runs on certain type of road, the response variance are derived for each index.

Road holding is described by the dynamic tire contact force. Recalling that the static tire load is $G = \left(m_1 + m_2 + m_3\right)g$ and that the dynamic tire load is $F_r = k_1\left(x_1 - x_0\right) + c_1\left(\dot{x}_1 - \dot{x}_0\right)$, the transfer function from $\dot{x}_0$ to $\hat{F}_r$ is

$$J_1 = \frac{\hat{F}_r}{G\hat{\dot{x}}_0} = \frac{k_1 + c_1 s}{sG}\left(\frac{\hat{x}_1}{\hat{x}_0} - 1\right) \tag{11}$$

The body suspension deflection is $f_s = x_2 - x_1$, and a transfer function from $\dot{x}_0$ is defined as

$$J_2 = \frac{\hat{f}_s}{\hat{\dot{x}}_0} = \frac{1}{s}\left(\frac{\hat{x}_2}{\hat{x}_0} - \frac{\hat{x}_1}{\hat{x}_0}\right) \tag{12}$$

The vertical vibration acceleration of the vehicle body (sprung mass) $\ddot{x}_2$ reflects vehicle ride comfort. A transfer function is defined as

$$J_3 = \frac{\hat{\ddot{x}}_2}{\hat{\dot{x}}_0} = s\frac{\hat{x}_2}{\hat{x}_0} \tag{13}$$

The motor suspension deflection is $f_m = x_3 - x_1$. A transfer function can be defined as

$$J_4 = \frac{\hat{f}_m}{\hat{\dot{x}}_0} = \frac{1}{s}\left(\frac{\hat{x}_3}{\hat{x}_0} - \frac{\hat{x}_1}{\hat{x}_0}\right) \tag{14}$$

The transfer function related to the motor vertical vibration acceleration varies with vehicle configurations. For a vehicle driven by fixed in-wheel motors, an index is derived as

$$J_5 = \frac{\hat{\ddot{x}}_1}{\hat{\dot{x}}_0} = s\frac{\hat{x}_1}{\hat{x}_0}, \tag{15}$$

For a vehicle driven by suspended in-wheel motors on the other hand, the index is defined as

$$J_5 = \frac{\hat{\ddot{x}}_3}{\hat{\dot{x}}_0} = s\frac{\hat{x}_3}{\hat{x}_0} \tag{16}$$

Eqs. (15) and (16) are related to the same index, but have different meaning. For a vehicle driven by a central motor drive, no such an index is defined.

For an easy understanding, these indexes will still be called in later sections as the tire force, suspension deflection, body acceleration, motor suspension deflection, and motor acceleration.

For each of the aforementioned indexes, the variance is also defined for each index by $(i = 1, 2, \cdots, 5)$

$$\sigma_{J_i}^2 = 4\pi^2 G_q(n_0) n_0^2 v \int_0^{\infty} \left|J_i(f)\right|^2 df \tag{17}$$

In the equation $n_0 = 0.1\,\mathrm{m}^{-1}$ is the spatial angular frequency as reference. The displacement spectral density $G_q\left(n_0\right)$ is used to characterize the road, and $G_q\left(n_0\right) = 64\ \times 10^{-6}\mathrm{m}^3$ for a B-class road [8]. v is the vehicle speed which is taken to be 10 m/s in later simulations, and f is time frequency.

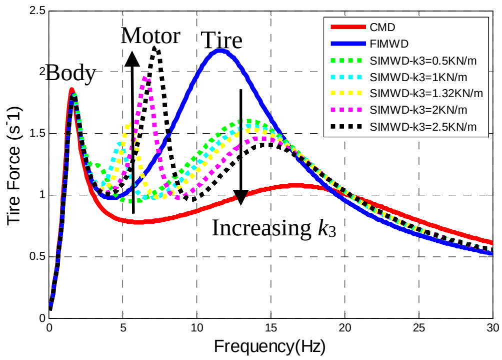
3 Sensitivity analysis

In this section, we conduct numerical simulations for influences of the motor system parameters on each of the aforementioned indexes. In other words, variation of the amplitude-frequency characteristics of each index is figured out by simulation with respect to parameters of m_3, k_3 and c_3. During the simulation, the value of one parameter varies while those of the remaining parameters remain constant. For each parameter, five different values are taken to reveal the influence. With all simulations, the prototype parameters of the vehicle are given in Table 1.

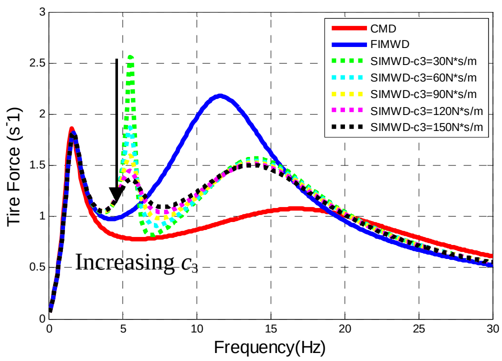
Table 1 Parameters of the vehicle models

	CMD	FIWMD	SIWMD
Unsprung mass m_1 (kg)	15.5	28	28
Sprung mass m_2 (kg)	224.5	202	202
Motor system mass m_3(kg)	/	10	10
Damping ratio c_2 (Ns/m)	1627.3	1627.3	1627.3
Damping ration c_3 (Ns/m)	/	/	90
Stiffness rate k_1 (N/m)	200000	200000	200000
Stiffness rate k_2 (N/m)	22366	22366	22366
Stiffness rate k_3 (N/m)	/	/	13200

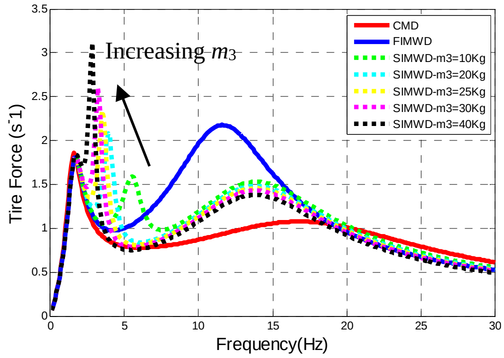
Figs. 2-6 present the calculated results of the vehicles with three different drive configurations. In the figures, there are three resonance peaks which correspond to vibrations respectively dominated by the body, the motor or tire.



(a) Sensitivity analysis on k_3



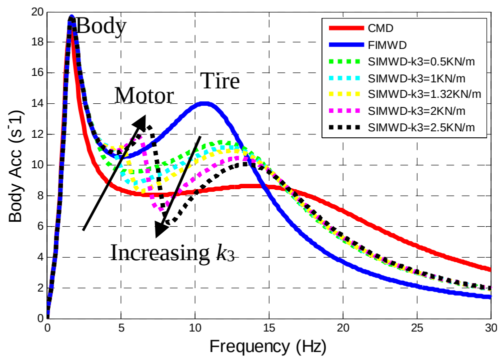
(b) Sensitivity analysis on c_3



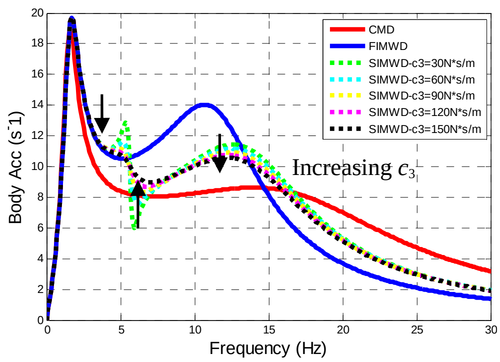
(c) Sensitivity analysis on m_3

Fig.2 Variation of J_1

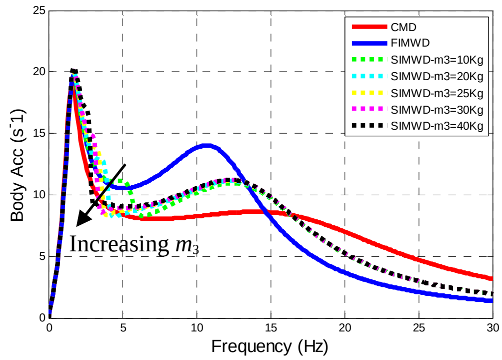
As shown in Fig.2, parameters k_3, c_3 and m_3 don't affect the tire contact force in a frequency band less than 3Hz which is dominated by vehicle body motion. The peak value of the motor resonance increases when k_3 and m_3 increase or c_3 reduces. The resonance vibration with vehicle suspension will increase if m_3 is not properly chosen, as the peak frequencies are so close to each other as shown in Fig.2c).



(a) Sensitivity analysis on k_3



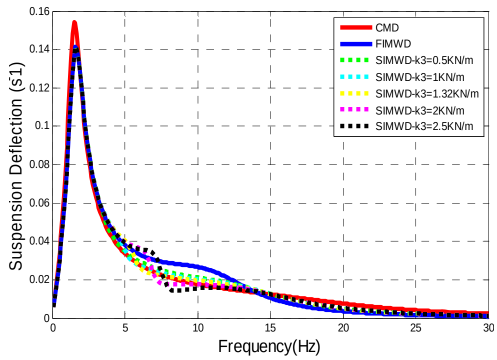
(b) Sensitivity analysis on c_3



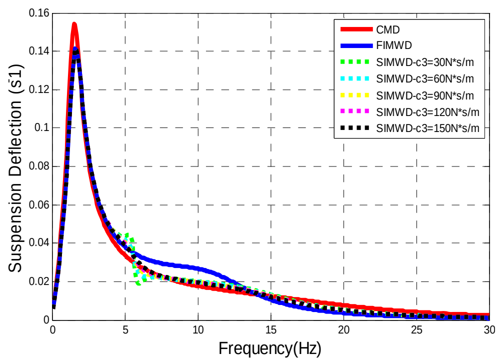
(c) Sensitivity analysis on m_3

Fig.3 Variation of J_3

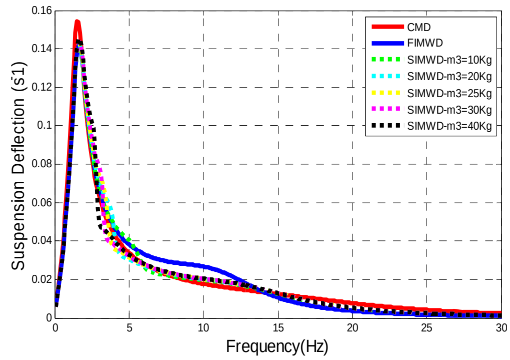
Looking at the relative body accleration as shown by J_3 in Fig.3, we can note that the peak value of the motor resonance clearly decrease with k_3 and c_3 ,but it is almost not affected by m_3.



(a) Sensitivity analysis on k_3

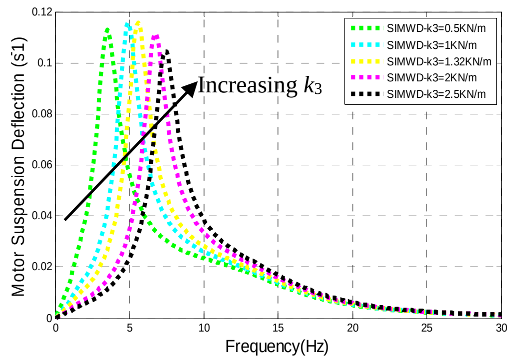


(b) Sensitivity analysis on c_3

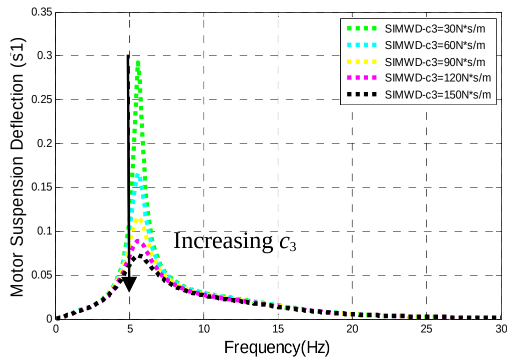


(c) Sensitivity analysis on m_3

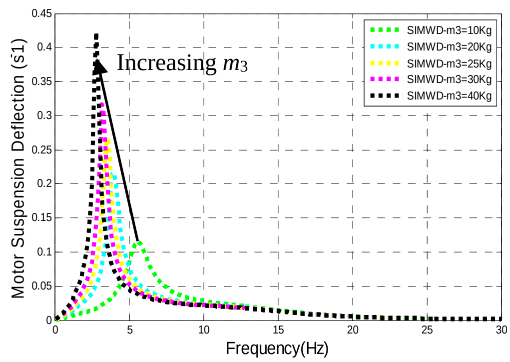
Fig.4 Variation of J_2



(a) Sensitivity analysis on k_3



(b) Sensitivity analysis on c_3

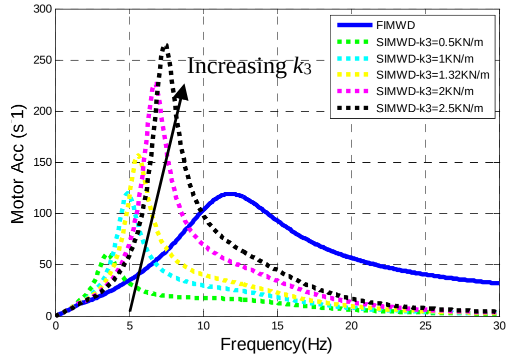


(c) Sensitivity analysis on m_3

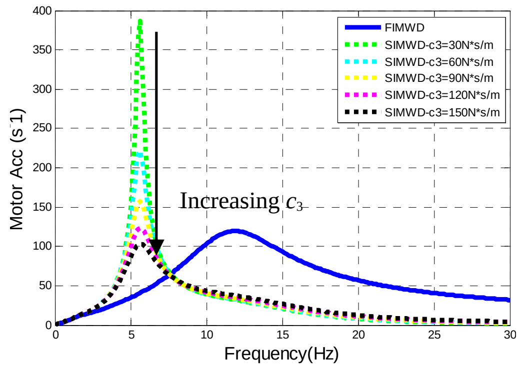
Fig.5 Variation of J_4

From Fig.4, it is noticed that the effect of parameters k_3 , c_3 and m_3 is negligible on the body suspension deflection.

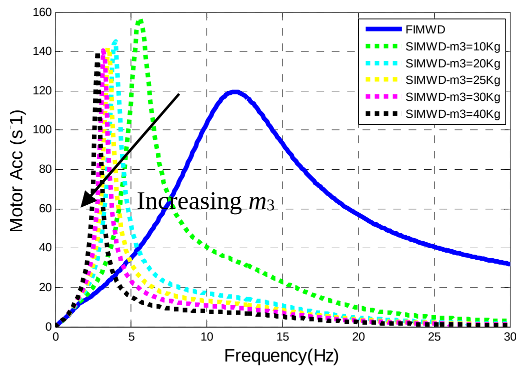
On the other hand, it can be found that the motor suspension deflection is clearly affected by parameters of k_3 , c_3 and m_3, as shown in Fig.5. As the deflection is very small however, it will not be included in the optimal objectives in Section 4.



(a) Sensitivity analysis on k_3



(b) Sensitivity analysis on c_3



(c) Sensitivity analysis on m_3

Fig.6 Variation of J_5

From Fig.6, it can be noted that the motor vibratory acceleration is at a magnitude of about 10^2. It implies a bad working condition for the motor, which deteriorates with k_3 . It is also observed that the resonance peak increases sharply in a very narrow frequency band if c_3 and m_3 increases to certain bigger values. Apparently, this sharp peak should be avoided to protect the motor from violent vibration.

4 Parameter optimization

Taking a model vehicle [4] as an example, optimizations are conducted on motor suspension to improve the performance of the vehicle and to reduce motor vibration. The objectives are defined as the tire contact force, the motor vibration acceleration, the body vibration acceleration or their combinations. We only optimize the parameters of the motor suspension system, namely, m_3, k_3 and c_3, while the rest parameters are kept unchanged. The initial values of the design variables, and their ponetial ranges defined by considering the wheel space and the motor mass, are listed in Table 2.

Table 2. Variables and their ranges

Variables	Initial values	Range
Mass of motor system m_3 (kg)	10	[10,20]
Damping of c_3 (Ns/m)	90	[0,inf]
Stiffness of k_3 (N/m)	13200	[10000,inf]

4.1 Single-objective optimization (SOO)

It is clear for a vehicle with suspended IWMs that the design of the motor suspension is a trade-off among various performances. It is still of great importance however, to firstly investigate potential improvement that can be achived by optimizatiom aiming at one performance each time.

Considering possible influence of the motor suspension, we define two objective functions for optimization. One, noted as A-Opt, is the area enclosed by the amplitude-frequency curve of a chosen performance index and the frequency axis, as shown in Eq. (18).

$$J_{\min}^{A} = \min\left\{\int_{f_0}^{f_t}\left|J_i\left(f\right)\right|df\right\}, i = 1,3,5 \quad (18)$$

Here f_0 and f_t are the lower and upper limits of the frequency range, respectively. In this example, they are separately chosen as 0.0Hz and 40.0Hz.

The other function, noted as P-Opt, is the summation of the peak values of a chosen index,

$$J_{\min}^{P} = \min\left(\sum_{k=1}^{n} P_k^{J_i(f)}\right) \quad (19)$$

In the equation, $P_k^{J_i(f)}$ is the k^{th} peak value of $J_i(f)$.

Using different indexes to construct the objective function, results in different optimized parameters for the motor suspension, as shown in Table 3.

Table 3 Parameters before and after optimizations

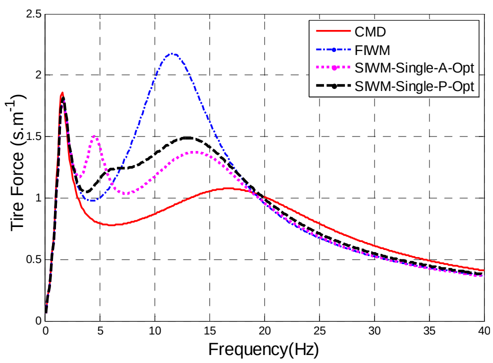
		k_3 (N/m)	c_3 (Ns/m)	m_3 (kg)
Initial value		13200.00	90.00	10.00
Min(J_1)	A-Opt	18110.76	275.11	20.00
	P-Opt	13248.91	282.00	10.00
Min (J_3)	A-Opt	136071.78	1547.70	20.00
	(P-Opt)	19042.00	410.07	10.00
Min(J_5)	(A-Opt)	10000.00	94.76	20.00
	(P-Opt)	10293.26	513.98	20.00

Using the suspension paramters from optimization, vehicle performaces are depicted in Table 4 and Fig.7.

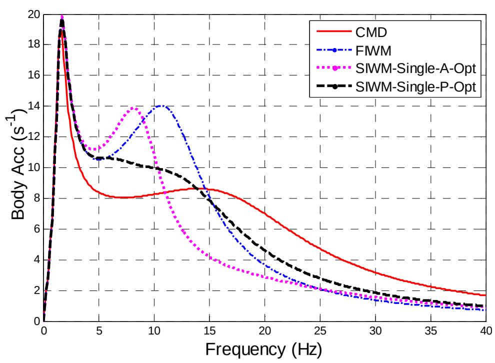
Table 4 Response variance comparison after SOO

	CMD	FIWMD	SIWMD		Enhancement (%)	
			A-Opt	P-Opt	A-Opt	P-Opt
J_1	0.0848	0.1142	0.0965	0.1007	15.5	11.8
J_3	0.7013	0.7851	0.7054	0.7200	10.2	8.30
J_5	/	5.9250	1.8851	2.0152	68.2	66.0

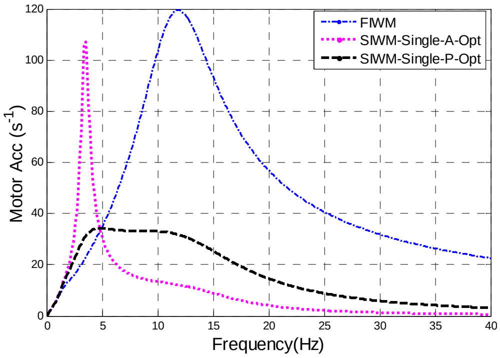
Note: 1) Enhancement is related to FIWMD;
2) '/' means not available.



(a) Tire force with parameters from min(J_1)



(b) Body vibration with parameters from min(J_3)



(c) Motor vibration with parameters from min(J_5)

Fig.7 Vehicle performances after SOO

From Fig. 7, one can note that the performances of all configurations render the vehicle of almost the same performances in the lower frequency band lower than 3Hz. The vehicle with in-wheel motor drive, either of fixed motor or suspended motor, performs better than the counterpart with a centered motor drive in a frequency band higher than 20Hz.

In the frequency range from 3Hz to 20Hz however, the vehicle with a center mass drive has much better performance than the other two configurations. Fortunately, the optimized motor suspension can grealty improve vehicle performance against the fixed in-wheel motor drive.

From the results, a comparison can also be made with the two optimization strategies. The A-Opt strategy shows more vairance reduction than the P-Opt strategy as shown in Table 3, and performs better in most of frequency band. On the other hand, the P-Opt strategy not only obviously reduces vibration level in the whole band, but also eliminates the resonant peak corresponding to the motor degree of freedom due to introduction of the motor suspension.

4.2 Multi-objective optimization (MOO)

Since there should be a compromise among various performances, it is valuable to explore the potential of a multi-objective optimization. To this end, a tentative endeavor is made by constructing an objective function as

$$J = w_1 \frac{J_1}{J_1^*} + w_3 \frac{J_3}{J_3^*} + w_5 \frac{J_5}{J_5^*} \tag{20}$$

In the equation, the constants of J_1^*, J_2^* and J_3^* are the optimal values obtained one by one from optimizations presented in subsection 4.1. The weighting factors w_i are roughly chosen as $w_1 = 0.5$, $w_3 = 0.4$ and $w_5 = 0.1$. Both the A-Opt and the P-Opt strategies are adopted for the multi-objective optimization. The parameters determined by optimizations are listed in Table 5.

Table 5 Initial and optimized parameters

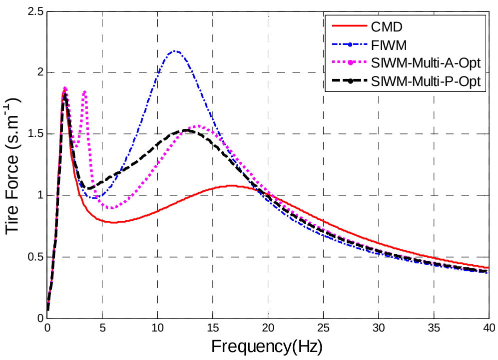
	k_3 (N/m)	c_3(Ns/m)	m_3(kg)
Initial value	13200.00	90.00	10.00
Optimized (A-Opt)	10000.00	99.49	20.00
Optimized (P-Opt)	10000.00	331.6	10.00

Using the optimized parameters for the motor suspension in the vehicle models, we get the vehicle performances as shown in Table 6 and Fig.8.

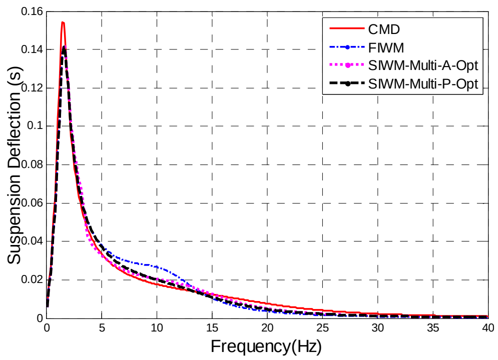
Table 6 Response variance comparison after MOO

	CMD	FIWMD	SIWMD		Enhancement (%)	
			A-Opt	P-Opt	A-Opt	P-Opt
J_1	0.0848	0.1142	0.1017	0.1012	11.0	11.4
J_2	0.0031	0.0031	0.0030	0.0030	2.04	2.17
J_3	0.7013	0.7851	0.7494	0.7306	4.55	6.94
J_4	/	/	0.0035	0.0014	/	/
J_5	/	5.9250	1.8519	2.8717	68.7	51.5

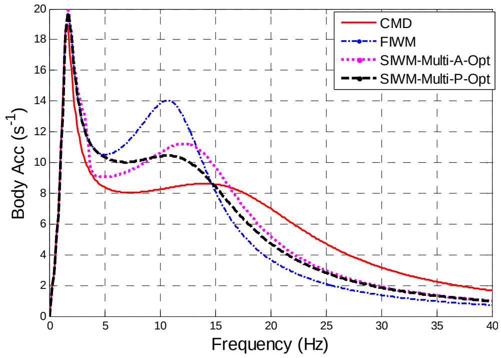
Note: 1) Enhancement is related to FIWMD;
2) '/' means not available.



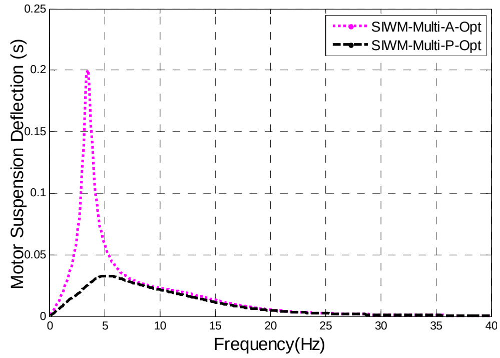
(a) J_1 with parameters from MOO



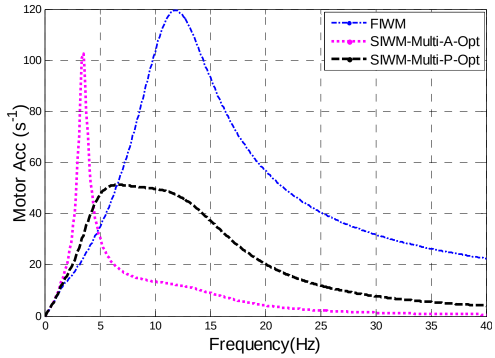
(b) J_2 with parameters from MOO



(c) J_3 with parameters from MOO



(d) J_4 with parameters from MOO



(e) J_5 with parameters from MOO

Fig.8 Vehicle performances after MOO

Observations can be made from the above results presented in Fig. 8 and Table 5.

At first, it can be noted that all the performance indexes can be simultaneously improved by the multi-objective optimization for vehicles with SIWMD.

Secondly, remarks about the two optimzation strategies given based on single-objective optimization still hold here for the multiple-objective optimization. For example, the P-Opt strategy can kick off the resonant peak due to the suspended motor from the amplitude-frequency curve of almost all indexes, as shown in Fig.8.

From simulation, we also notice that the response variance value cannot be used independently to evaluate vehicle performance, as it just reflects the average level. Instead, it should be used by a simultaneous review with the amplitude-frequency curves of the idexes.

5 Summary

Vertical performance of electrical vehicles driven by suspended in-wheel motors are studied in this paper through dynamic simulation on quarter-car models. The concerned performace are related to tire road holding, vehicle ride comfort and motor vibration. Sensitivity analysis is performed on the indexes with respect to the motor mass and motor suspension parameters. Extensive optimizations are then conducted on motor suspension to explore the potential in improving the vehicle performances. During the optimization, each of the index or a combination of the indexes is employed to construct the objective function according to two strategies. One strategy concerns the average amplitude of one index or a combination of the indexes in the whole frequency domain, and the other one focuses on the curve peaks of one index or the combination of the indexes. From the analysis, conclusions can be drawn as follows.

1) For a vehicle driven by fixed in-wheel motors, increased unsprung mass not only deteriorate tire road holding and vehicle ride comfort, but also causes more violent motor vibration compared with a vehicle driven by a centered motor. Violent vibration implies possibility of premature damage of the motor and adjacent structures.

2) The above problems can be effectively compressed by introducing properly designed suspensions to support the in-wheel motors.

3) A new degree of freedom is resulted from motor suspension, adding new resonant peak in the amplitude - frequency curve of the most of the performance indexes. The peak can be dismissed by parameter optimization using one strategy to construct the objective function.

4) More endeavor should be made to improve the performance of vehicles driven by in-wheel motors.

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Authors

Wei TONG was born in Mianyang, China. He graduated from Hunan University, China, in 2010 with B.S. degree in Automotive Engineering. And he is currently a postgraduate student in the Department of Automotive Engineering at Tsinghua University, China. His research work focuses on vehicle vibration and dynamics.

Zhichao HOU is a Professor in Department of Automotive Engineering at Tsinghua University. He received a B.Sc.(Eng.) and a M.Sc. (Eng.) in Mechanical Engineering from Harbin Shipbuilding Engineering Institute in 1988 and 1991, respectively. In1995 he received a Ph.D. degree in Applied Mechanics from Tsinghua University. His current research interests include structural dynamics and fatigue, vibration control and lightweight design with a background of vehicle NVH and ride comfort.