EVS26
Los Angeles, California, May 6 - 9, 2012

Optimal Battery Sizes for Plug-In-Hybrid Electric Vehicles

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Abstract

Plug-in-hybrid electric vehicles (PHEV) have recently been introduced into markets world-wide. They offer large shares of cheap electric driving without big expensive batteries. However, the question which battery size is cost optimal for a certain driver or group of drivers with their specific driving profiles is difficult to answer in general. Here, we derive explicit formulae for the optimal battery size for any driving profile and parameters such as fuel and battery costs by analytically minimizing the total cost of ownership for the driver(s). Our results are applied to realistic individual driving profiles as well as the distribution of German driving profiles (from a large-scale survey) and are also found to be in good agreement with full numerical optimization of the same driving profiles. With the right-sized batteries, PHEVs can capture market shares more easily. The present results can be directly applied to find the optimal PHEV battery size for any other country's population or group of drivers.

Keywords: battery, cost, PHEV (plug in hybrid electric vehicle), EREV (extended range electric vehicle)

1 Introduction

The introduction of electric vehicles (EVs) can help to reduce green house gas emissions from the transport sector as well as local emissions [1, 2]. In addition, electric propulsion is more efficient than propulsion via internal combustion engines and can further aid to support the shift from oil to other energy sources. However, a substantial obstacle for consumers might be the limited range of full electric vehicles [3]. Possible solutions embrace frequent stops for recharging or the inclusion of an additional internal combustion engine as “range extender”. Electric cars with a “range extender”, known as plug-in hybrids (PHEVs), are advantageous from the customers point of view: Consumers can travel long distances if necessary and have a high electric driving share when traveling shorter distances [4] (which is cheap and efficient). Furthermore, PHEVs can be designed with smaller batteries than battery electric vehicles, which cover shorter regular trips, and thus avoid an oversized and expensive battery.

Since the battery represents an important investment, the question about the optimal battery size to choose arises from the consumer's and manufacturer's perspective (cf. [5–7]). The former wants to optimize their total costs (typically modeled as total costs of ownership) whereas the latter is motivated to design vehicles fitting customer needs. Thus, manufacturers face the problem to find standardized or 'typical' battery capacities for vehicle construction. An additional problem arises from the large spectrum of vehicle usage patterns, including different trip length distributions, duration and location of stops between trips. A usage-specific design of battery capacities for the same car could also be useful. For the total costs of ownership and in absence of ubiquitous charging infrastructure (being the case presently), the driving behavior can be summarized in driving profiles. Here, we find the optimal PHEV battery size for arbitrary driving profiles. The structure of the present paper is as follows. We will state a mathematical formulation of the problem and derive analytical solutions for the optimal PHEV battery size in sec. 2. Applications of the results for an exemplary driving profile and the average German driving trip length distribution will be discussed in sec. 3. We will compare our results for the latter case with a numerical optimization in sec. 4 and will end with a short summary.

2 Analytical Results

2.1 Derivation

To find the cost optimal battery size κ^*, we consider the total costs of ownership (TCO) for PHEVs [1,2] denoted as $C_{\text{tot}}(\kappa)$ and given by the sum of purchase and usage costs

$$C_{\text{tot}}(\kappa) = (I+c_B\kappa)a_n(p)+\sum_l w_l C(r_l,\kappa). \quad (1)$$

Here, I denotes the purchase costs of the PHEV (excluding the battery), c_B the specific battery cost (in Euro/kWh), $a_n(p) = \frac{p(1+p)^n}{(1+p)^n-1}$ is an annuity factor for n years at an interest rate p. The second term in eq. (1) represents the variable costs of going on a trip l of length r_l with costs $C(r_l,\kappa)$ per trip. That is,

$$C(r,\kappa) = \begin{cases} c_E r & \text{for } r \leq L_0(\kappa) \\ c_E L_0(\kappa) + c_R(r - L_0(\kappa)) & \text{for } r > L_0(\kappa) \end{cases}$$

are the variable costs of a single trip of length r with a PHEV with a battery of size κ and an electric driving range $L_0(\kappa) = \kappa/e_E$ (e_E is the specific electric energy consumption in kWh/km). Each trip has a frequency of occurrence of w_l during one year. The driving costs are determined by the electric driving range $L_0(\kappa)$ and the specific electric (range extender) driving costs c_E (c_R) in Euro/km.

Our goal is to determine the most cost-effective battery size. Mathematically, this is an optimization problem. Since the dependence of the total costs of ownership is given as a function of the battery size in eq. (1) we only need to take the derivative with respect to the battery size κ and solve the resulting equation for the optimal battery size κ^*. Dividing additionally by $\sum_l w_l$ leads to the following equation (using $v_l = w_l/\sum_l w_l$, the normalized probability for trip l):

$$\sum_l v_l \Theta(r_l - L_0(\kappa)) = \frac{e_E c_B a_n(p)}{(c_R - c_E)\sum_l w_l} \equiv f_0 \quad (2)$$

where $\Theta(x) = 0$ for $x < 0$ and $\Theta(x) = 1$ for $x \geq 0$ denotes the unit step function. This equation needs to be solved for the optimal battery size κ^*. Note that κ- dependence enters only via the electric driving range $L_0 = \kappa/e_E$ and that the right hand side of eq. (2), abbreviated as f_0, is independent of κ.

The equation to be solved for κ, eq. (2), has a clear structure: the left hand side is a normalized sum of unit step functions and thus a step-wise non-negative function $f(\kappa)$ with $f(\kappa) \leq 1 \forall \kappa$. The right hand side is a simple constant. Subtracting f_0 on both sides of eq. (2), we need to find the zeros of $f(\kappa) - f_0$. The specific value of f_0 changes the zero between different steps of $f(\kappa)$. Thus the general solution to the equation will inherit the step-wise structure from the left hand side of eq. (2), $f(\kappa)$. The general solution giving the cost optimal battery size for discrete driving profiles is given by

$$\kappa^* = e_E \sum_n r_n \Theta\Big[\Big(f_0 - \Big(1 - \frac{v_n}{2} - \sum_{m<n} v_m\Big)\Big)^2 - \Big(\frac{v_n}{2}\Big)^2\Big]. \quad (3)$$

The result shows that the optimal battery size κ^* is the battery size allowing the longest still cost-effective trip to be included in full electric driving. The mentioned step-wise structure of the solution is evident. This solution can also be understood by studying the expression for the total costs of ownership (cf. eq. (1)), directly. For a discrete driving profile, these costs are a sum of terms that are linear in κ or independent thereof. The above equation, eq. (2), can be generalized to continuous driving profiles

$$\int_0^\infty v(r)\Theta(r-\kappa/e_E)\mathrm{d}r = \frac{e_E c_B a_n(p)}{(c_R - c_E)\int w(r)\mathrm{d}r}.$$

Let $V(r) = \int_0^r v(r')\mathrm{d}r'$ denote the anti-derivative to $v(r)$ and using the properties of the unit step function $\Theta(-x) = 1 - \Theta(x)$, we find

$$V(\kappa/e_E) = \lim_{r\to\infty} V(r) - \frac{e_E c_B a_n(p)}{(c_R - c_E)\int w(r)dr}.$$

The inverse $V^{-1}(r)$ to $V(r)$ allows us to write the result explicitly (where the normalization of $v(r)$ implies $\lim_{r\to\infty} V(r) = 1$)

$$\kappa^* = e_E \cdot V^{-1}\left[1 - \frac{c_B}{(c_R - c_E)}\frac{e_E a_n(p)}{\int w(r)dr}\right]. \quad (4)$$

The two expressions, eq.s (3) and (4), for the optimal PHEV battery capacity are explicit analytical solutions for the most cost-effective (minimizing[1] the total cost of ownership) battery size for any discrete or continuous driving profile. For the rest of the paper, we will apply these solutions to specific driving profiles and compare the results to numerical simulations of optimal battery sizes for individual driving profiles.

2.2 Discussion

The two main results of the previous section, eq.s (3) and (4), require further discussion. An important property of both solutions is that they do not depend on the important parameters battery price and fuel (electric or gasoline) consumption price separately, but instead on a combination of energy and battery prices

$$x = \frac{c_B}{c_R - c_E}. \tag{5}$$

It is intuitively expected that the positive effect of a low battery price on the total costs of ownership can be compensated by lowering fuel consumption prices, i.e. the difference between gasoline consumption costs and electric driving costs increases. Our results demonstrate this effect explicitly.

The results for the optimal battery sizes are quite general in scope: They can be applied to arbitrary (discrete or continuous) PHEV driving profiles. Specific trips can be incorporated with their respective frequency of occurrence. This might be the driving profile of an individual or a large fleet of vehicles (where the limit of a continuous distribution is a valid approximation), e.g. for driving profiles of a country's vehicle fleet. In the latter case, the solution for large fleets, eq. (4) might be interesting for vehicle manufacturers in supporting a decision about the best battery size to choose in designing PHEVs. Furthermore it is important to mention that the present solution gives *one* optimal solution for a given set or distribution of trips. It cannot directly give a distribution of solutions (such as displayed in the numerical results in Figure 3). However, extensions of the results of eq.s (3) and (4) are possible. One could, e.g., use a set of log-normal distributions with distributions for mean and variance $\mathcal{P}(\mu, \sigma)$ leading to a distribution of optimal battery sizes $\mathcal{P}'(\kappa)$. Another possibility would be to use distributions of individual trip length distributions, e.g. a linear combination of a Gaussian and a random background (as discussed in [11]). Both are beyond the scope of the present paper.

[1] Strictly speaking, we have only found stationary points (vanishing first derivative) but not shown that the TCO are actually minimal. This would require $\mathrm{d}^2 C_{tot}(\kappa)/\mathrm{d}\kappa^2 > 0$. However, in all examples below it will be clear that the solutions are actually minimizing the TCO.

3 Application to Specific Driving Profiles

Let us discuss simple examples of discrete driving profiles to clarify the meaning of the general solution eq. (3). The simplest case of only one trip length has the solution $\kappa^* = e_E r \Theta(1 - f_0)$. More generally, if the driving profile consists of a short and a long trip (each with their respective frequency of occurrence) the optimal battery capacity will be either zero (when batteries are too expensive) or giving an electric driving range that coincides with one of the trip lengths (depending on the specific battery costs). According to the general solution and possibly counter-intuitive, it is never cost-effective to buy a battery that allows slightly longer electric driving than required for the shortest cost-effective distance. Buying a slightly bigger battery does not pay off. Using Feynman's problem solving algorithm [10], this reasoning can be made explicit and the solution to the case of two trip lengths can also be given explicitly

$$\kappa^* = \begin{cases} 0 & 1 < f_0 < \infty \\ e_E r_1 \text{ for} & 1 - v_1 < f_0 < 1 \\ e_E r_2 & 0 \le f_0 < 1 - v_1 \end{cases}.$$

This can also be expressed as (the products of step functions are required to fulfill two conditions at the same time)

$$\kappa^* = e_E r_1 \Theta[1 - f_0]\Theta[f_0 - (1 - v_1)] + e_E r_2 \Theta[(1 - v_1) - f_0]\Theta[f_0 - (1 - v_1 - v_2)].$$

Using $\Theta(a - x)\Theta(x - a) = \Theta(x^2 - a^2)$, this simple example can be generalized to more complex driving profiles and one obtains the solution given in the previous section, eq. (3).

Using the large scale survey of German mobility "Mobility panel" [8] conducted annually and containing more than 20,000 trips per year, one can extract the distribution of trip lengths per vehicle and day $v(r)$. Figure 1 shows the cumulative distribution $\mathrm{CDF}(r) = V(r)$ for the trip lengths (averaged over 1994 – 2008) and a log-normal fit

$$V(r) = \int_0^r v(s)\mathrm{d}s = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\mu - \ln r}{\sqrt{2}\sigma}\right)\right], \tag{6}$$

where $\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)\mathrm{d}t$ denotes the error function. The fitting parameters obtained

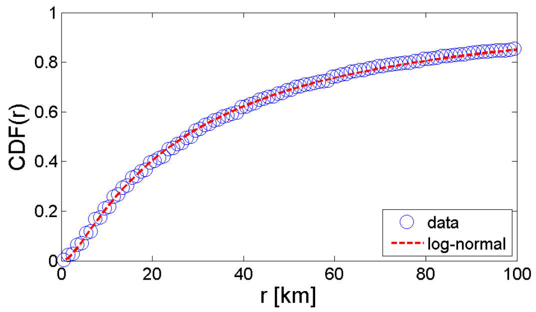

Figure 1: Cumulative distribution function of daily motor vehicle trip lengths for German driving. Data is based on [8] and comprise over 20,000 single trips during one week.

from a least square fit are $\mu = 3.21$ and $\sigma = 1.15$. With this data, the optimal battery size of PHEVs for the German market can be easily computed using eq. (4).

Figure 2 shows the optimal PHEV battery size for the average German driving profile as a function of battery cost for the parameters $e_E = 0.2$ kWh/km, $c_R = 0.06$ Euro/km, $a_n(p) = 0.13$. The result from eq. (4) is shown for different values of the electric driving consumption costs c_E. We observe the existence of a critical battery price c_B^* below which the investment in a battery with capacity larger than zero turns economically justified. For $c_E = 0.04$ Euro/km, the critical $c_B^* \approx 480$ Euro/kWh (the green curve in Figure 2). With decreasing battery costs, the optimal battery size clearly increases, i.e. the larger investment pays off for the driving profile under consideration. For battery costs around 200 Euro/kWh the optimal battery size is in the range of $13 - 18$ kWh depending on the costs of electric driving. The result can be interpreted as follows: for the average German distribution of trip lengths and battery costs of around 200 Euro/kWh a battery size of the order of 10 kWh would be cost optimal when the electric driving costs are about 3 EuroCent/kWh.

4 Comparison to Numerical Results

In [9], Kley and co-workers reported numerical results on optimal PHEV battery sizes based on simulating the most cost-effective battery size for a large set of individual driving profiles. We shortly summarize their appraoch in the following, further details and numerical parameters can be found in [9].

Based on reported start and end times as well as the trip length, driving profiles were aggregated for each individual in the data [8]. At the arrival of each trip, the type of parking location (at home, at work, elsewhere) is determined from the trip purpose. The resulting driving and parking profiles had a total length of seven days. With multiple time-variant parameters such as electricity price, or different power connections, the optimal battery profile was derived by solving a minimization problem. The battery state of charge was simulated as decision variable in each interval, describing the consumers search for a cost-optimal charging strategy. Depending on battery prices and infrastructure, different battery sizes are optimal for different shares of drivers (according to their specific driving profiles). The results of the simulation are shown in Figure 3. The results have been computed for different infrastructure scenarios (private charging only, private and semi-public, private and semi-public and public charging; left to right panel). This cannot be directly included in the analytical result eq. (4) since the driving profile only contains information about trip lengths but not on the vehicle's position during breaks or the waiting time between subsequent stops. This would be required for a full comparison with the numerical results in Figure 3. In this respect, the numerical simulations are superior. For the following comparison, we focus on the left panel of Figure 3. The results in Figure 3 show that the average optimal battery size is slowly increasing for a decreasing battery price. Only for very low battery costs below 200 Euro/kWh, very large batteries become economically reasonable. Both is in full agreement with the results from our analytical approach (cf. Figure 2).

For a quantitative comparison, we study the average optimal battery capacities that have been

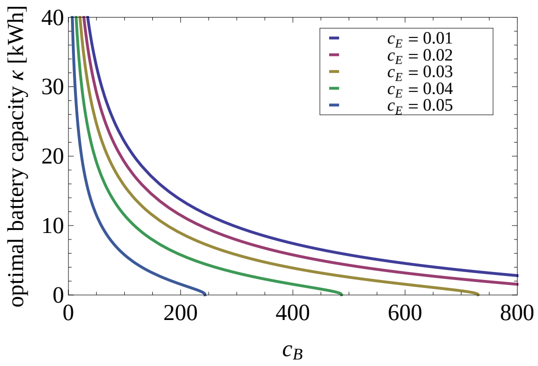

Figure 2: Optimal battery size κ^* for PHEVs (with the average German driving profile diplayed in Figure 1) as a function of specific battery costs (in Euro/kWh) for different specific electric driving costs c_E (in Euro/km). Parameters: $e_E = 0.2$ kWh/km, $c_R = 0.06$ Euro/km, $a_n(p) = 0.13$.

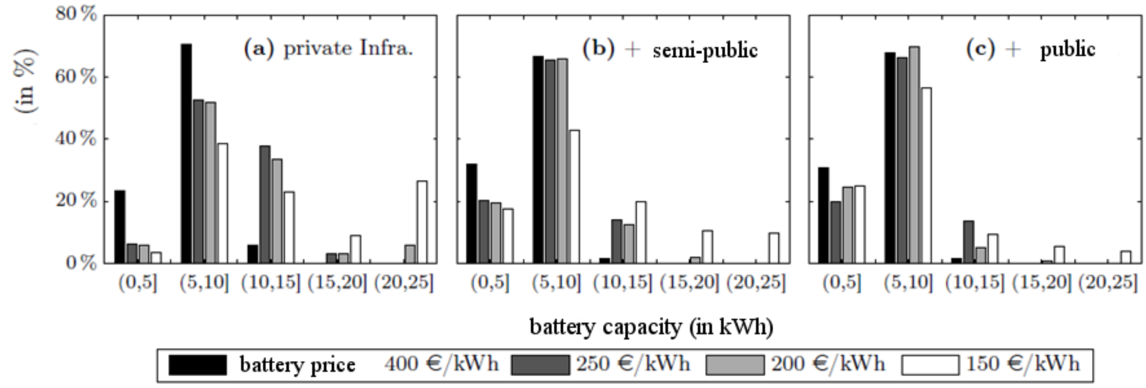

Figure 3: Optimal PHEV battery capacities from numerical optimization of individual driving profiles. Shown are optimal PHEV battery sizes for different charging infrastructures (left to right panel: private charging only, private and semi-public, private and semi-public and public charging) and different battery prices (black: 400 Euro/kWh, dark gray: 250 Euro/kWh, light gray: 200 Euro/kWh, white: 150 Euro/kWh). Taken from [9] (further parameters given therein).

numerically using the same setup [4]. We use the same parameters as in [4] to compute optimal battery capacities analytically using eq.s (4) and (6). The parameters read $c_E = 0.036$ Euro/km, $c_R = 0.084$ Euro/km, $n = 12$, $p = 0\%$, and $e_E = 0.2$ kWh/km. Both results are summarized in Table 1. The results from the numerical and analytical approach shown in Table 1 are in good qualitative agreement. Yet they do not match exactly but show deviations of the order of up to 25%. However, full agreement could not be expected: the numerical simulation finds the optimal battery for an individual driving profile and then averages over all optimal battery capacities compared to the analytical approach averaging first over all driving profiles and computing a single optimal battery afterwards. These two results, an average of optima and the optimum for an average, do not necessarily need to be identical. In summary, within the scope of the approach presented (evaluation of average driving profiles without infrastructure information) we find good qualitative agreement between the numerical and analytical results.

Table 1: Summary of results for optimal battery capacity κ^*

Battery price [Euro/kWh]	Numerical κ^* [4] [kWh]	analytical κ^* [kWh]
400	7.3	7.0
250	10.0	11.3
200	10.6	13.6
150	14.0	16.9

5 Conclusion

We determined the optimal battery size of PHEVs for any driving profile (for an individual or an entire population) and given cost parameters by analytical minimization of the total costs of ownership. Our results can be directly applied to determine the cost-optimal battery for several markets or driving behaviors of users. From an automotive manufacturer's perspective, our results require a scalable battery set-up and indicate that customers need a better understanding of their driving behavior, e.g. by measuring driving profiles, in order to choose the cheapest battery size for them. To be more realistic, further research should include different vehicle sizes and charging infrastructures.

Acknowledgement

The authors would like to thank Martin Wietschel and David Dallinger for stimulating discussions.

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