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Optimal Control of a Plug-In Hybrid Electric Vehicle (PHEV) Based on Driving Patterns

Namwook Kim[1], Daeheung Lee[1], Suk Won Cha[1], Huei Peng[2]
[1]*School of Mechanical and Aerospace Engineering, 314-311, Seoul National Univ. Daehak-Dong, Kwanak-Gu, Seoul, South Korea, alde1@snu.ac.kr*
[2]*Departement of Mechanical Engineering, University of Michigan, USA*

Abstract

In this study, Pontryagin's minimum principle (PMP) is applied to obtain the control law of plug-in hybrid vehicles. The results show that the minimization of the equivalent fuel consumption with a pre-defined weighting coefficient, which is a costate of PMP, is an effective way to obtain a control strategy that minimizes the overall energy. Dynamic programming yields results that are close to being global optimal. To realize the control algorithm solved by PMP, we introduce an adaptive concept that is based on driving patterns, and we conclude that an instantaneous optimal strategy with a properly selected costate is sufficiently simple for application to a real-time controller and is a desirable strategy that yields satisfactory results.

Keywords: optimization; PHEV; energy consumption; power management; simulation.

1 Introduction

With regard to environmentally friendly strategies, electric vehicles (EVs) are possibly considered as next-generation vehicles because they do not produce emissions and the electrical devices in the EVs have high energy efficiencies. However, before EVs can be successfully applied, several problems await solutions. One of the problems is that a battery in an EV has a small energy density, which means the EV cannot drive for a long while without other energy resources. Therefore, the Plug-in Hybrid Electric Vehicle (PHEV) can be an alternative solution until breakthroughs are found for designing new sources of electric energy that have high energy densities. In general, though it has not been commercialized by major automotive companies, a PHEV has a huge battery, which is directly charged by an external plug so that the PHEV can drive longer in a pure electric mode than a general hybrid electric vehicle (HEV). In that PHEVs is supposed to be similar to EVs at short-term driving, the energy management strategy is different with the strategy of HEV whereas it should be similar with HEV in a long-term drive way. With regard to the optimal control for energy management, various control concepts for hybrid electric vehicles have been widely applied to solve the power management problem. In the context of optimal control, Dynamic Programming (DP) and Pontryagin's Minimum Principle (PMP) are two different approaches to obtain optimal trajectories for deterministic optimal control problems. In the minimum fuel consumption problem of HEVs, the DP method guarantees a global optimal solution by detecting all possible control options [1], [2], [3]. On the other hand, the control problem can be simple when PMP is used because the only control parameter we have to consider is a costate of PMP. In this paper, we will demonstrate an application of PMP in the control problem of PHEVs as an alternative method that guarantees solutions that are close to being global optimal.

2 A static model in a backward simulator

We can consider two different types of simulator, a forward type and a backward type. The difference between these two is that, in the backward simulation, we can calculate the input control that satisfies the vehicle performance; so, the simulator can test all possible input controls. On the other hand, the forward-type simulator calculates the vehicle behaviour from the single input control whereas we can apply more realistic conditions to the forward type than to the backward type. In this paper, we use a backward-type simulator to obtain optimal control trajectories. A static model is considered in the simulator. The static model has a drawback because the transient dynamics of power resources are not considered in the model. However, the model is appropriate for the backward simulator because it is tractable and can be used to test all possible inputs.

2.1 Vehicle model

The split hybrid system shown in Fig. 1 is used as the target system in this paper, which models the Toyota Hybrid System (THS) by using a single planetary gear as the power split device [4]. The engine, motor-generator 1 (MG1), and motor-generator 2 (MG2) are attached to the carrier gear, sun gear, and ring gear of the planetary gear, while the vehicle is attached to the ring gear through a final drive.

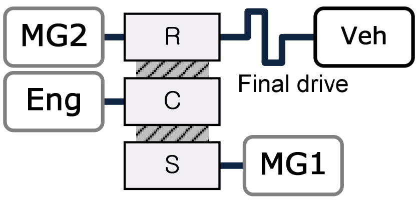

Fig. 1. The power split hybrid system, which has a power split device.

In the static model, the relationships among the engine torque, the motor-generator torque, and the output torque can be obtained through the lever analogy.

$$T_{eng} + T_{mg1} + T_{mg2} - 1/\zeta \cdot T_{req} = 0 \qquad (1)$$

$$T_{eng} + (1+R)T_{mg1} = 0 \qquad (2)$$

In Eqs. (1) and (2), T_{eng} , T_{mg1} , T_{mg2} , and T_{req} are the torques of the engine, MG1, MG2, and the requested output torque of the transmission, respectively. Further, R and ζ are the gear ratio of the planetary gear set and the final gear ratio, respectively. The relationships among the speeds of the power resources can be also obtained by the lever analogy, which is expressed as follows.

$$S_{mg1} = (1+R)\cdot\left(S_{eng} - \zeta \cdot S_{req}\right) + \zeta \cdot S_{req} \qquad (3)$$

$$S_{mg2} = \zeta \cdot S_{req} \qquad (4)$$

In Eqs. (3) and (4), S_{eng} , S_{mg1} , S_{mg2} , and S_{req} are the speeds of the engine, MG1, MG2, and the requested output speed. To obtain the instantaneous optimal operating line, we choose the engine torque and the engine speed, T_{eng} and S_{eng} , as the control variables. From (1)~(4), we can obtain the operating torques and speeds of MG1 and MG2, which are functions of the control variables when the requested output torque and speed are given by a driving cycle.

$$\begin{bmatrix} T_{mg1} \\ T_{mg2} \end{bmatrix} = \frac{1}{-(1+R)} \begin{bmatrix} 0 & 1 \\ 1+R & R \end{bmatrix} \cdot \begin{bmatrix} -1/\zeta \cdot T_{req} \\ T_{eng} \end{bmatrix} \qquad (5)$$

$$\begin{bmatrix} S_{mg1} \\ S_{mg2} \end{bmatrix} = \begin{bmatrix} -R & 1+R \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \zeta \cdot S_{req} \\ S_{eng} \end{bmatrix} \qquad (6)$$

Based on the operating values of MGs, the required power of the battery can then be calculated as follows.

$$P_{bat} = \eta_{c1}{}^{k} \cdot T_{mg1} \cdot S_{mg1} + \eta_{c2}{}^{k} \cdot T_{mg2} \cdot S_{mg2} \qquad (7)$$

In Eq. (7), the efficiencies of MG1 and MG2, viz., η_{c1} and η_{c2} , are numerically calculated by interpolating motor-efficiency maps of each MG, which include the motor's loss and the inverter's loss. Further,

$$k = \begin{cases} 1, & recuperating \\ -1, & motoring \end{cases} \qquad (8)$$

Finally, the derivative of SOC , namely, $\dot{SOC}$, can be calculated from the battery power and the current SOC.

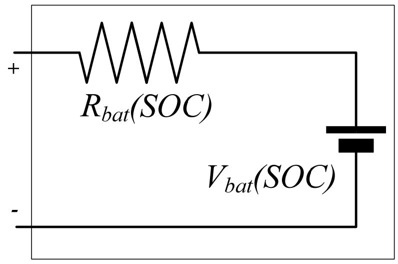

Fig. 2. An equivalent circuit model of a battery with an internal resistance and an output voltage, which are functions of the state of charge.

Considering the equivalent open circuit voltage and internal resistance in Fig. 2, $S\dot{O}C$ is a function of SOC and P_{bat}, which can be expressed as:

$$S\dot{O}C = \frac{1}{Q_{bat}} \cdot \frac{V_{bat} - \sqrt{V_{bat}^{\ 2} - 4R_{bat}P_{bat}}}{2R_{bat}} \tag{9}$$

In Eq. (9), Q_{bat} is the capacity of the battery, R_{bat} is the internal resistance of the battery, and V_{bat} is the output voltage of the battery. Finally, the derivative of SOC is a function of T_{eng}, S_{eng}, and SOC, as per (5) ~ (9).

$$S\dot{O}C = f\left(T_{eng}, S_{eng}, SOC\right) \tag{10}$$

We can use a fuel consumption rate map to obtain the numerical value of $\dot{m}_{fc}$ because the consumption rate, $\dot{m}_{fc}$, is a function of the control variables, T_{eng} and S_{eng}.

$$\dot{m}_{fc} = L\left(T_{eng}, S_{eng}\right) \tag{11}$$

In conclusion, $S\dot{O}C$ in (10) and $\dot{m}_{fc}$ in (11) are numerical functions of $T_{eng}, S_{eng}, S_{eng}$, and SOC, when the requested output condition is specified. The parameters of the vehicle are from the Prius04 model in Powertrain System Analysis Toolkit (PSAT), which are summarized in Table 1 .

Table 1. Vehicle parameters used in simulations.

Vehicle total mass	1405 kg
Engine	Si_1497_57_US_04Prius
Motor1	pm_25_50_prius
Motor2	pm_15_30_prius
Battery	Nimh_6_168_panasonic_MY04_Prius
Planetary gear ratio	2.6 (78/30)
Final gear ratio	4.113
Transmission gear efficiency	90 %
Rolling resistance coefficient	$0.007 + \left(0.00012 \times \text{vehicle velocity}\right)$
Frontal area	1.746 m^2
Drag coefficient	0.29
Wheel radius	0.305 m
Air density	1.23 kg/m^2

2.2 Local optimal operating points

The optimal control of PHEVs should be solved for the whole horizon time, whereas the candidates of the optimal control can be instantaneously calculated in the static model. The Confined-Optimal Operating Line (C-OOL) was introduced in [13]; it is the best engine operating line for the relevant battery power, when the output speed and torque are specified. To obtain the C-OOL, we calculate possible engine operation points wherein the same electric energy is either charged or discharged. Then, the point of the minimum fuel consumption rate out of the possible points can be selected as an optimal candidate. The C-OOL is a family of these optimal candidates, as shown in Fig. 3.

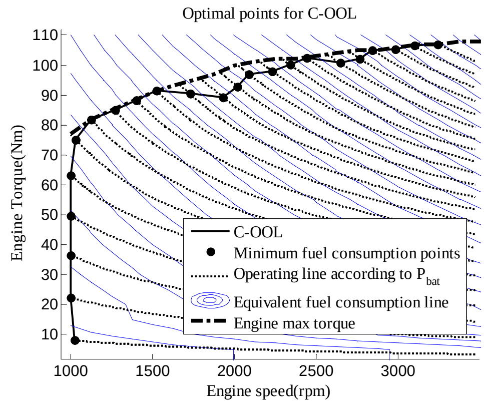

Fig. 3 An example of the C-OOL, when $T_{req} = 100Nm$ and $S_{req} = 100rad/s$. The dotted lines are feasible lines of operation for specific P_{bat}. The resolution of P_{bat} in the figure is 1.5kW, whereas it is 0.05kW in our simulation.

Now, we can present the best fuel consumption rate, $\dot{m}_{fc}$, as a function of P_{bat} (see Fig. 4). The best fuel consumption function in Fig. 4 clearly implies that we can save more fuel if we use more electric energy.

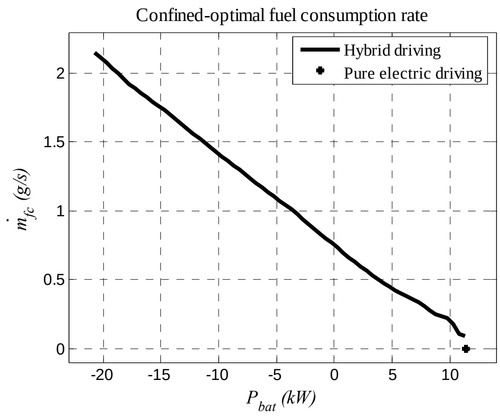

Fig. 4. Instantaneous optimal fuel consumption rate line in the domain when $T_{req} = 100Nm$ and $S_{req} = 100rad/s$.

From the viewpoint of horizon optimal control, this optimal process reduces the dimensions of the control variables, by which we do not consider the inferior engine operation points when solving the problem in the horizon plane, whereas we need to calculate the C-OOL at each and every second. The physical meaning of the line in Fig. 4 is simple: given that the engine always operates at the best point, less fuel needs to be consumed when more battery power is used, and vice versa. In general, the requested output torque and speed vary with time; hence, we can say, with reference to the time-horizon plane, that the fuel consumption rate, $\dot{m}_{fc}$, is just a function of P_{bat} and t, as in (12).

$$\dot{m}_{fc} = g\left(P_{bat}, t\right) \tag{12}$$

Additionally, the pure electric driving point, namely, the point to the right in Fig. 4, is an operating point at which the battery supplies all the energy that is needed to drive the vehicle, while the engine does not operate or – if appropriate – operates at optimal speed with no fuel consumption but with engine drag.

3 Optimal control of PHEVs

Minimization of fuel economy is the only object in this paper though there are other criteria that we can consider, such as the acceleration performance or emission level. The optimal control problem of PHEVs can be solved by optimization techniques that are based on optimal control theories. In this chapter, we discuss the general solution for deterministic optimal control problems and introduce Pontryagin's Minimum Principle (PMP) as a method of solving the control problem for PHEVs.

3.1 Optimal control theory

There are two representative approaches to solve deterministic optimal control problems. One is Dynamic Programming (DP), which pursues Bellman's principle of optimality, and the other is trajectory optimization that is based on Pontryagin's Minimum Principle (PMP).

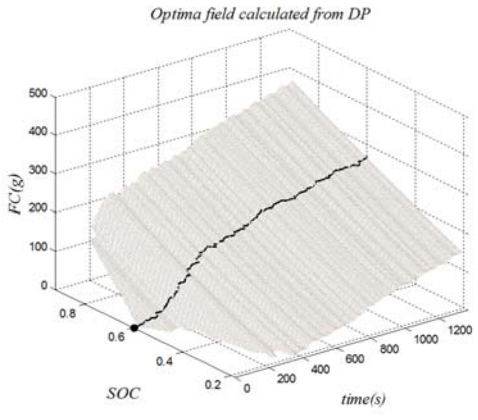

Fig. 5. An example of an optimal field that is calculated from DP with a forward-looking type refers to the total fuel consumption from the origin to the current point.

In general, DP calculates the optimal field, which is a family of optimal cost-to-go. (See Fig. 5 although it shows a cost-from-start.) On the other hand, PMP produces the necessary conditions that optimal trajectories must satisfy, which means it does not guarantee optimality whereas a solution that is yielded by DP is always an absolute or global optimum for deterministic problems.

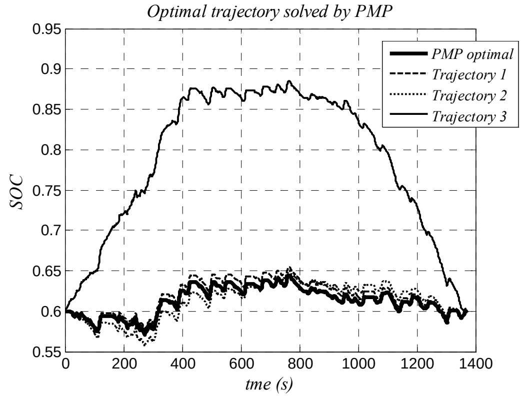

Fig. 6. An optimal SOC trajectory that is solved by PMP. PMP just guarantees that the solution is superior to trajectories that are near the solution.

Mathematically, these two methods, DP and PMP, can be linked when the costate of PMP is interpreted as an instantaneous sensitivity between the state and the optimal field on the optimal trajectory [11]. This means that PMP checks for optimality only on the optimal trajectory, and DP needs far more calculations to check all the possible trajectories. In conclusion, DP generates a superior solution than PMP whereas PMP requires less computing time to obtain the solution. However, PMP has an advantage in that the solution can be calculated instantaneously if we know an appropriate costate, which will be described in the following section.

3.2 Optimal control in PHEVs

Assuming that the main goal of the control strategy is to coordinate the operations of the three power resources to minimize the overall energy consumption, a PMP problem can be formulated as:

$$\begin{cases} \min\left\{J = \int_{t_0}^{t_f} g\left(P_{bat}\left(t\right), t\right) dt\right\} \\ \text{subject to:} \\ \qquad S\dot{O}C\left(t\right) \quad = f\left(SOC\left(t\right), P_{bat}\left(t\right)\right) \\ \qquad SOC\left(t_0\right) \quad = SOC\left(t_f\right) \\ \qquad SOC_{\min} \le SOC\left(t\right) \le SOC_{\max} \\ \qquad P_{\min} \le P_{bat}\left(t\right) \le P_{\max} \end{cases} \tag{13}$$

In the above, $g(P_{bat},t)$ is the fuel consumption rate, as in (12), and $f(SOC,P_{bat})$ is the state equation for SOC, as in (9). Further, the state variable, SOC, and the control variable, P_{bat}, are limited by $SOC_{\min}$, $SOC_{\max}$, $P_{\min}$, and $P_{\max}$. An optimal control variable is the variable that minimizes a Hamiltonian function at every time step, where the Hamiltonian is defined as:

$$H = g\left(P_{bat}(t),t\right) + p(t)\cdot f\left(P_{bat}(t),SOC(t)\right) \quad (14)$$

In (14), $p(t)$ is the costate function in PMP [12]. Based on the necessary conditions, the optimal control, P_{bat}^*, can be calculated as

$$P_{bat}^* = \arg\min_{P_{bat}} H\left(P_{bat}(t), p^*(t), SOC^*(t), t\right) \quad (15)$$

and the costate equation is defined as

$$\dot{p} = -\partial H \big/ \partial\left(SOC\right) \quad (16)$$

The optimal trajectory should satisfy the necessary conditions, such as: the state equation in (9); the co-state equation in (16); and the condition expressed in (15). Furthermore, the optimal trajectory also satisfies the boundary condition in (13) whereby the final SOC arrives at the desired final value, $SOC\left(t_f\right)$. The Equivalent Consumption Minimization Strategy (ECMS) is substantially linked to PMP given that equivalent consumption in ECMS is defined similar to the Hamiltonian in Eq. (14) [6], [7], [8]. The similarity of these two techniques is described in [13].

3.3 Optimality of the PMP method

PMP produces the necessary conditions for optimality whereas DP (or the Hamilton-Jacobi-Bellman equation) guarantees global optimality [9], [10]. Therefore, the PMP solution might be inferior to the solution from DP, and the performance degradation may not be insignificant in the general optimal control problem. However, the optimal trajectory that is yielded by PMP is near the global optimal solution in a hybrid electric vehicle system. The state equation, $f\left(P_{bat}(t), SOC(t)\right)$, is a function of P_{bat} and SOC. (See Fig. 7.)

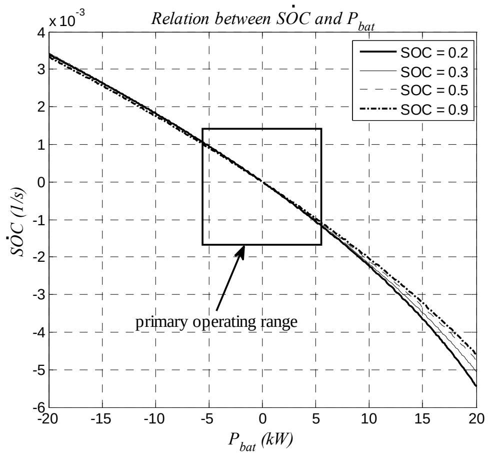

Fig. 7. The time derivative of SOC is influenced by the SOC but the influence is not significant in the primary operating range, viz., -5kW to +5kW.

It is, however, not the case that SOC definitely influences $S\dot{O}C$ especially in the primary operating range of the battery because the resistance and the voltage of the battery are hardly influenced by SOC, which means that we can consider $S\dot{O}C$ to be a function of just P_{bat} and expressed as

$$S\dot{O}C(t) = f\left(P_{bat}(t)\right) \quad (17)$$

Given that the state equation is a function of only P_{bat}, the costate is constant because both sides of Eq. (16) are zero. In that case, the optimal trajectory that satisfies the necessary conditions of PMP is unique and the uniqueness guarantees the global optimality of the solution, which is described in [13]. Furthermore, the constant costate makes the optimal control law simple enough to be implemented.

3.4 Optimal control simulator

We developed a new simulator, OC_SIM, which can solve the optimal control problem of PHEVs. The simulator features several types of hybrid electric vehicle, and we can select various combinations to test the fuel economies of the vehicles.

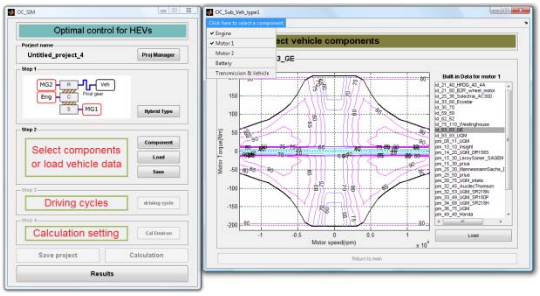

Fig. 8. The main panel and a component panel in OC_SIM.

The simulator can execute both optimal control techniques, DP and PMP, by which a user simulates hybrid electric vehicles at various driving schedules.

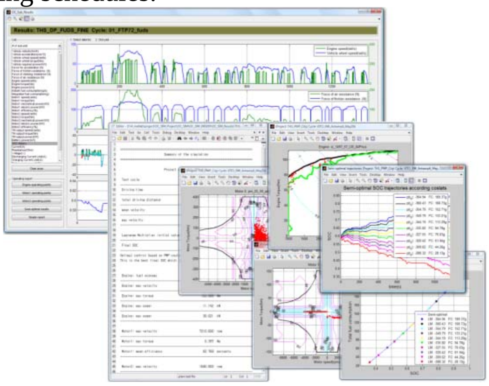

Fig. 9. A panel that displays results and a simple report of the simulation results.

A results panel presents all the results of the optimal simulation by which the user can evaluate operating points of power resources. The summary of the simulation is reported by a text file. Furthermore, there is a function by which users can check the influence of the co-state of PMP if the user selects the PMP method. The simulator, OC_SIM, can be downloaded from **Error! Reference source not found.**.

4 Optimal control for the PHEV

As stated in section 3.3, the necessary conditions of PMP generate a global optimal solution under the assumption that the battery resistance and voltage are independent of *SOC*. In this section, we calculate and compare the optimal solutions that are yielded by: ⅰ) DP; ⅱ) PMP without the above assumption of independence; and ⅲ) PMP with that assumption. On the basis of the simulation results, we can conclude that the solution is near-optimal if we use a constant co-state under the assumption of independence. We also propose a method to guess an appropriate co-state on the basis of the driving patterns.

4.1 Simulation results

Using the OC_SIM, we calculate optimal control solutions for the target vehicle at FTP72.

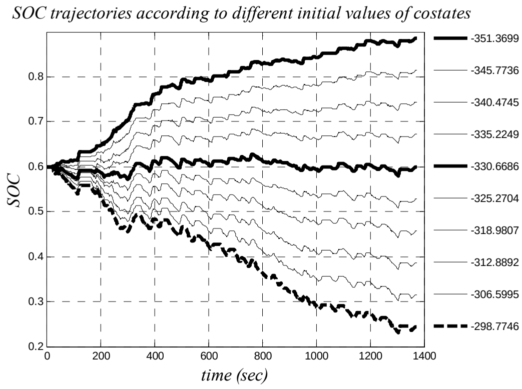

Fig. 10. SOC trajectories that are solved through PMP with different initial co-states at FTP72.

Fig. 10 shows the results that are derived from PMP for different initial co-states, in which a larger initial costate yields a lower final *SOC*. Given that the costate can be interpreted as a parameter that is equivalent to the electric usage and the fuel consumption, it is natural that the total usage of electric energy is influenced by the costate. To obtain the optimal control trajectory for PHEVs, we set the initial *SOC* as 0.6 and the final *SOC* as 0.2. Then, we found that the optimal initial value of the costate was -301.1 in the FTP72 cycle. The fuel economy of the control trajectory that is derived from PMP is close to the global optimal results that are yielded by DP. (see Table 2).

Table 2. Optimal fuel economies for PHEVs under different techniques.

Method	DP	PMP	
		Exact solution $(p(0) = -301.1)$	Constant co-state $(p(0) = -293.4)$
FE (km/l)	65.716	65.621	65.358

In Table 2, there is an additional case when a constant costate is used instead of a variable costate for an exact solution. Though a constant costate is available only under the assumption of independence, the fuel economy in this case is also close to that under the global optimal solution. The *SOC* trajectories of the two cases seem to be slightly different but the engine operation patterns in Fig. 12 show that these two controls are similar.

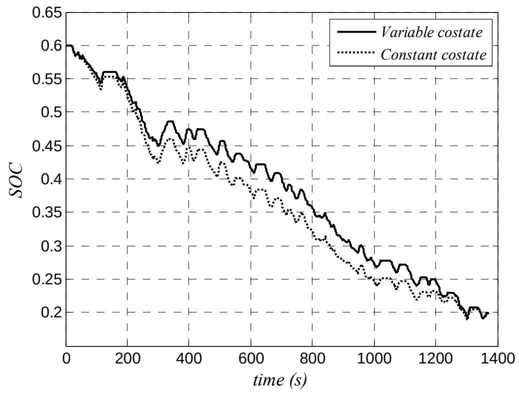

Fig. 11. Two optimal trajectories that are derived from PMP. One is calculated with an exact solution with a variable co-state, while the other is obtained by a constant co-state under the assumption of independence for the battery.

Hence, the fuel economy under a constant costate is close to the fuel economy under the exact solution.

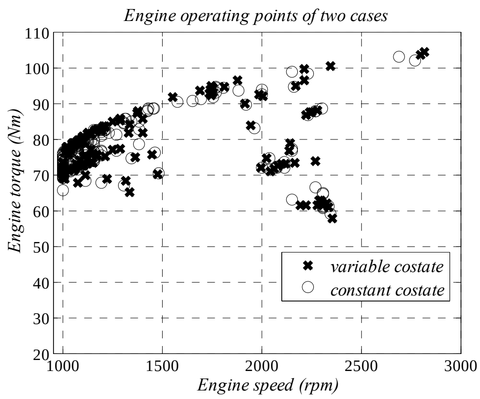

Fig. 12. Engine operation points of the two cases show similar tendencies.

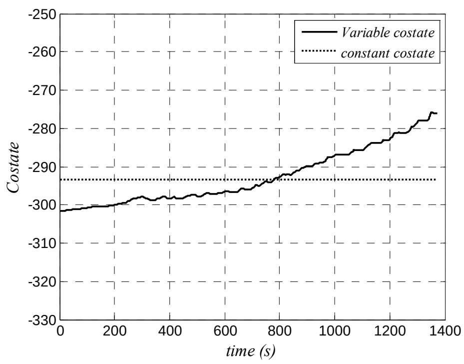

Fig. 13. The comparison of the two costates, a variable co-state under the exact solution and a constant costate under the assumption of independence for the battery.

The use of a constant costate not only reduces the computational burden but also makes it easy to guess an appropriate costate from driving patterns. Unfortunately, the difference between the constant costate and the exact co-state in PHEVs is over 5% whereas the difference in HEVs is at most 1% [13] because PHEVs use a wider range of the battery than HEVs. However, by noting the results on the fuel economies, we can conclude that the optimal control that is based on PMP with a constant co-state is still a viable compromise that simplifies the problem.

4.2 Approximation model

The costate in PMP, which is interpreted as the weighting coefficient for the equivalent consumption of battery energy, determines how fast electric energy is used during a driving cycle. Therefore, it is essential to select an appropriate costate that makes the SOC trajectory be a global optimal solution. From an observation of several simulation results [14], the optimal costate is found to be closely related to the patterns of a driving cycle that are represented by the effective SOC drop rate and the effective mean power over the duration of traction. The effective SOC drop rate is defined as

$$S\dot{O}C_{eff} = \frac{\Delta SOC_{eff}}{\Delta t_{eff}} \tag{18}$$

In (18), Δt_{eff} is the total traction time when a powertrain produces a propulsion force and ΔSOC_{eff} is the total variation of the SOC during Δt_{eff}. Further, the effective mean power is defined as

$$P_{mean} = \frac{\sum P_{eff}}{\Delta t_{eff}} \tag{19}$$

In (19), P_{eff} is the requested power for t_{eff}. With these two parameters, we obtain an equation that describes the optimal costate, p_{opt}, for PHEV control, which is expressed as

$$\begin{aligned} p_{opt} &= -364.5 - 75190 \cdot S\dot{O}C_{eff} + 2.49 \cdot P_{mean} \\ &\quad +3317 \cdot S\dot{O}C_{eff} \cdot P_{mean} \end{aligned} \tag{20}$$

Form Eq. (20), we can calculate an optimal co-state, which is shown in Fig. 14. To obtain the $S\dot{O}C_{eff}$, we calculate the equivalent value of the parameter from the summation of the power that is requested during recuperation and the difference between the initial SOC and the final (target) SOC.

$$\dot{SOC}_{eff} = \left\{ \left(C_{con} \cdot \sum P_{recup} \right) - 0.4 \right\} / \Delta t_{eff} \quad (21)$$

In (21), P_{recup} is the recuperating power and C_{con} is a conversion coefficient that is determined by the battery size, which is 1.161×10^{-4} for our PHEV.

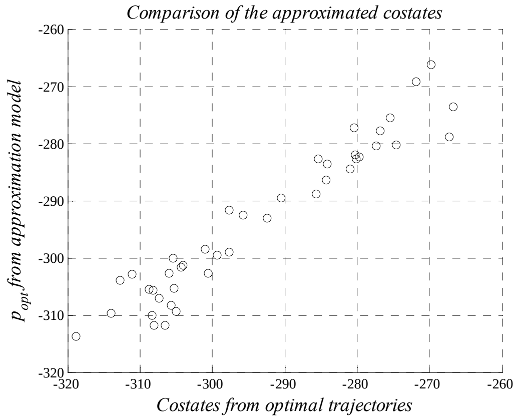

Fig. 14. Optimal vs. calculated co-states from Eq. (20). The calculated co-states with regard to the driving pattern are close to the optimal co-states that are yielded by PMP.

Fig. 14 shows that the approximated costates that are based on Eq. (20) are close to the optimal costates that are yielded by PMP. This means that the optimal costates can be estimated from the two parameters, once the future driving schedule is specified. Unfortunately, it is not possible for us to know the future driving schedule without external devices such as navigation systems that are based on GPS. It is, however, possible to estimate these two parameters from prior driving records if the PHEV is used in regular driving patterns, e.g., daily commuting. In conclusion, a control concept that is based on PMP is an efficient method that can be applied to a real-time controller because the costate, which is an assumed value that nevertheless can be obtained from driving patterns, is the only parameter we have to consider. At the same time, the method yields good results with regard to fuel minimization for PHEVs.

5 Conclusion

The optimal control that is based on Pontryagin's minimum principle (PMP) possibly possesses the potential for application in real-time energy management strategies because it can achieve near-optimal control of the power resources. Further, we can instantaneously control the system under optimality. The only parameter we have to consider in PMP, viz., the costate, is a parameter that is related to regulating the final SOC . The costate is influenced by driving schedules but we can estimate an appropriate costate from an approximation model with two representative parameters, $\dot{SOC}_{eff}$ and P_{mean}. These two parameters might be calculated from past driving patterns when the driving patterns of vehicles are repeated, as with daily commuting. The co-state approximation model that is based on pattern recognition parameters in our study is adequate for the control of a PHEV in that the PHEV can be used in daily commuting rather than in long-duration driving on highways. Our study focuses on an adaptive concept to decide an appropriate costate that is based on the approximation model.

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Namwook Kim, Ph.D. Student,
School of Mechanical and Aerospace Engineering,
Seoul National University,
alde1@snu.ac.kr

Daeheung Lee, Ph.D. Student,
School of Mechanical and Aerospace Engineering,
Seoul National University,
ideal02@snu.ac.kr

Suk Won Cha, Assistant professor,
School of Mechanical and Aerospace Engineering,
Seoul National University,
cwcha@snu.ac.kr

Huei Peng, Professor,
Department of Mehchanical Engineering,
University of Michigan,
hpeng@umich.edu