

## Power Characteristics for Power Split Type HEV System

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### Abstract

Power characteristics of the power split type HEV are investigated using the lever analysis. For the analysis, generalized expression of the 4-node lever is derived to represent the torque and speed relationship. From the lever relationship, the state equations of the input split, output split and compound split configurations are derived. Using the state equations, torque, speed, and power characteristics of the engine, motor MG1 and MG2 are investigated. It is expected that the simulation results of the power characteristics for the input, output and compound split configuration can be used in design of the power split type HEV with smaller motor size and lower fuel consumption while satisfying the demanded vehicle performance.

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*Keywords:* Hybrid electric vehicle, Power split hybrid, Planetary gear, Lever analysis

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### 1 Introduction

In hybrid electric vehicle(HEV), transmission plays an essential role which distributes the internal combustion engine(ICE) power to the wheels and the energy storage unit or combines the ICE power and the motor power. Recently, power split type transmission has been used in many HEVs, such as Toyota, Ford, Nissan and GM. The power split type configuration has both mechanical and electrical path combining the planetary gear set and two motors. The electrical path is able to provide continuously variable transmission operation using the two motors[1]. In addition, this configuration offers electric-only capability, and seamless capability to decouple the road load demand from the engine in a manner similar to that in a series hybrid. Since the engine is decoupled from the wheels, the engine can be operated on its best efficiency region meanwhile the mechanical path transmits the power with high efficiency compared with

the series hybrid. These characteristics enable improvement in fuel consumption and reduction in emissions. However, in spite of these advantages, the power circulation which may occur along the closed loop and relatively large two motors have been mentioned as major drawbacks. To overcome these problems, various power split configurations have been investigated.

The power split configurations can be classified as three types: (1)input split, (2)output split and (3)compound split[2]. Each power split configuration has its own characteristics.

In this paper, power characteristics of various power split configurations are investigated by lever approach and are compared in terms of the torque, speed and power for the EV, HEV and regenerative braking mode.

## 2 Torque and speed equation of power split system using lever analysis

In Figure 1, a lever model is shown for a general power split HEV. In this study,  $i$ ,  $a$  and  $b$  are introduced as a relative lever distance[2].  $i$  is the lever distance from the output to the engine,  $a$  is the lever distance from the output to MG1(motor generator 1) and  $b$  is the lever distance from the output to MG2. The engine torque  $T_e$ , MG1 torque  $T_{MG1}$  and MG2 torque  $T_{MG2}$  are applied as the input. The output torque  $T_{out}$  is considered as being applied at the node where the vehicle mass is connected. The lever model in Figure 1 is a 2DOF(degree of freedom) system where two(2) speeds can be determined if the other two(2) speeds are given. Even if there is an inertia at every node, the inertia effects of MG1 and MG2 are neglected since they can be considered small enough compared with those of the engine and the vehicle mass. From the lever model in Figure 1, dynamic equations of the power split system can be derived as

$$T_{out} = \frac{R_t}{N_f} (M \dot{V} + F_{load}) \quad (1)$$

$$T_{out}i + T_{MG1}(a-i) + T_{MG2}(b-i) = 0 \quad (2)$$

$$(T_e - J_e \dot{\omega}_e)i + T_{MG1}a + T_{MG2}b = 0 \quad (3)$$

where  $R_t$  is the tire radius,  $N_f$  is the final reduction gear ratio,  $M$  is the vehicle mass,  $V$  is vehicle velocity,  $F_{load}$  is the road load,  $J_e$  is the engine inertia and  $\omega_e$  is the engine speed.

Representing Eq. (1)~(3) in matrix form, the following state space equation can be obtained.

$$\begin{bmatrix} \dot{\omega}_e \\ \dot{V} \end{bmatrix} = \begin{bmatrix} \frac{1}{J_e} & \frac{a}{J_e i} & \frac{b}{J_e i} & 0 \\ 0 & \frac{N_f}{R_t M} - \frac{a N_f}{R_t M i} & \frac{N_f}{R_t M} - \frac{b N_f}{R_t M i} & -\frac{1}{M} \end{bmatrix} \begin{bmatrix} T_e \\ T_{MG1} \\ T_{MG2} \\ F_{load} \end{bmatrix} \quad (4)$$

The output speed  $\omega_{out}$ , MG1 speed  $\omega_{MG1}$  and MG2 speed  $\omega_{MG2}$  can be obtained from the engine speed  $\omega_e$  and the vehicle velocity  $V$  as

$$\omega_{out} = V \frac{N_f}{R_t} \quad (5)$$

$$\omega_{MG1} = \frac{a}{i} \omega_{engine} + (1 - \frac{a}{i}) \omega_{out} \quad (6)$$

$$\omega_{MG2} = \frac{b}{i} \omega_{engine} + (1 - \frac{b}{i}) \omega_{out} \quad (7)$$

Using the dynamic equations of the general power split system, the power characteristics of the input split, output split and compound split system (Figure 2) are investigated. The vehicle parameters used in the simulations are shown in Table 1[3].

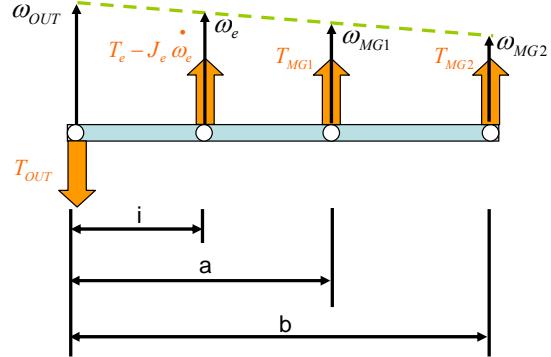


Figure 1 Lever model of general power split HEV system

Table 1 Parameters and constraints of power split HEV

| Parameters                        |                                   |
|-----------------------------------|-----------------------------------|
| Final reduction gear ratio, $N_f$ | 4.1                               |
| Tire radius, $R_t$                | 0.3 m                             |
| Engine inertia, $J_e$             | 0.1 kg·m <sup>2</sup>             |
| Vehicle mass, $M$                 | 1500 kg                           |
| Constraints                       |                                   |
| MG1                               | 30 kW<br>Maximum Speed 10,000 RPM |
| MG2                               | 30 kW<br>Maximum Speed 10,000 RPM |
| Battery                           | 20 kW                             |

In the input split system(Figure 2a), MG2 is directly connected to the output and the engine is connected to MG1 through the planetary gear. Assuming that  $i=-1$ ,  $a=3$  and  $b=0$  in Figure 1, the state equation can be obtained as

$$\frac{d}{dt} \begin{bmatrix} \omega_e \\ V \end{bmatrix} = \begin{bmatrix} 10 & 30 & 0 & 0 \\ 0 & -0.0182 & 0.0091 & -0.0007 \end{bmatrix} \begin{bmatrix} T_e \\ T_{MG1} \\ T_{MG2} \\ F_{load} \end{bmatrix} \quad (8)$$

It is found from the Eq. (8) that  $T_{MG2}$  does not affect the engine speed. Instead,  $T_{MG2}$  can be used to propel the vehicle directly without using the engine. It is noted from Eq. (8) that the electric vehicle(EV) mode can be implemented by operating the motor MG2.

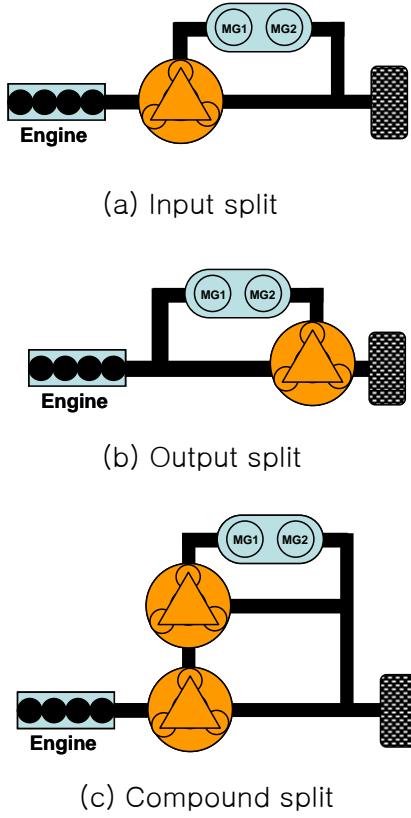


Figure 2 Power split HEV

In the output split system (Figure 2b), the engine is directly connected to MG1 and the input power is split through the planetary gear located at the output side. Assuming that  $i=-1$ ,  $a=-1$  and  $b=-3$  in Figure 1, the state equation can be obtained as

$$\frac{d}{dt} \begin{bmatrix} \omega_e \\ V \end{bmatrix} = \begin{bmatrix} 10 & 10 & 30 & 0 \\ 0 & 0 & -0.0182 & -0.0007 \end{bmatrix} \begin{bmatrix} T_e \\ T_{MG1} \\ T_{MG2} \\ F_{load} \end{bmatrix} \quad (9)$$

It is seen from Eq. (9) that  $T_{MG1}$  does not affect the vehicle speed, but only influences on the engine speed.

In the compound split system (Figure 2c), the motor MG1 and MG2 are not directly connected to the engine nor the output. Assuming that  $i=1$ ,  $a=1.5$  and  $b=-3$  in Figure 1, the state equation can be obtained as

$$\frac{d}{dt} \begin{bmatrix} \omega_e \\ V \end{bmatrix} = \begin{bmatrix} 10 & 15 & -30 & 0 \\ 0 & -0.0046 & 0.0364 & -0.0007 \end{bmatrix} \begin{bmatrix} T_e \\ T_{MG1} \\ T_{MG2} \\ F_{load} \end{bmatrix} \quad (10)$$

It is noted from Eq. (10) that both  $T_{MG1}$  and  $T_{MG2}$  affect the engine speed as well as the vehicle speed.

### 3 Power characteristics of the power split HEV system

Using the state equations of various power split structures, power characteristics are investigated for electric vehicle (EV) mode, hybrid electric vehicle (HEV) mode and regenerative braking (REGEN) mode. In the simulation, it is assumed that the engine is operated at the arbitrary optimal operating point,  $\omega_e=200\text{rad/sec}$ ,  $T_e=100\text{Nm}$ .

#### 3.1 EV mode

##### 3.1.1 Input split

Figure 3 shows the simulation results of the input split HEV for EV mode. For the input split configuration, since the vehicle is propelled by the MG2 that is connected to the output driveshaft, the output torque  $T_{out}$  is equal to  $T_{MG2}$ , and  $T_{MG1}$  is zero.  $\omega_{MG2}$  increases in the positive direction while  $\omega_{MG1}$  decreases in the negative direction. From Figure 3c, it is seen that the output power  $P_{out}$  is equal to  $P_{MG2}$ .

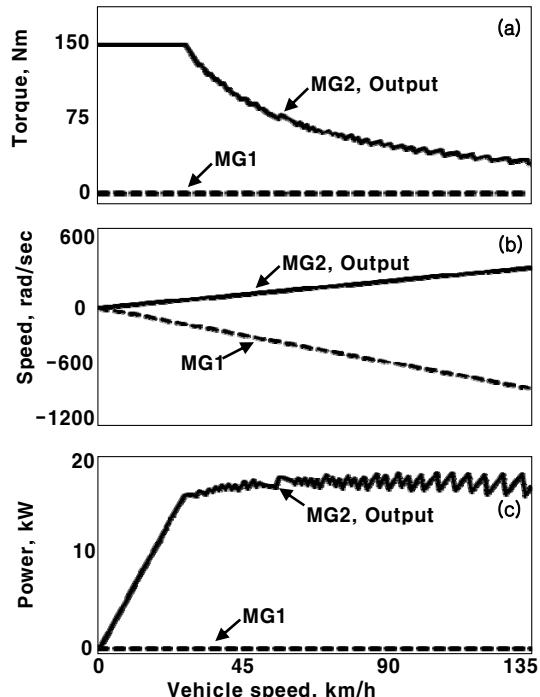


Figure 3 Input split HEV for EV mode

##### 3.1.2 Output split

For the output split system,  $T_{MG2}$  (Figure 4a) shows the negative value to propel the vehicle meanwhile  $T_{MG1}$  generates the positive value from the torque balance of the planetary gear.  $\omega_{MG1}$  is zero since the engine is off in the EV mode, and  $\omega_{MG2}$  decreases in the negative direction. The vehicle

output torque increases up to  $V=45\text{kph}$  and maintains almost constant power(Figure 4c). Even if  $P_{MG1}$  is zero due to the zero velocity of the MG1, it is found that the MG1 plays as a brake, and needs to be controlled to provide the required torque in the EV mode.

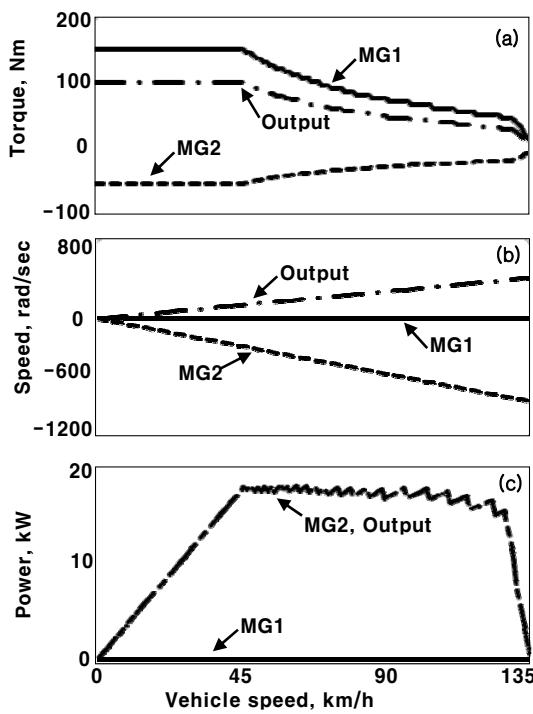


Figure 4 Output split HEV for EV mode

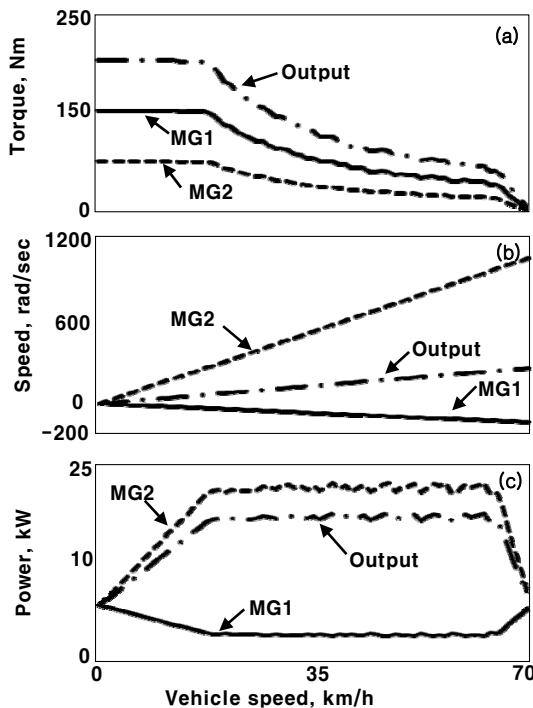


Figure 5 Compound split HEV for EV mode

### 3.1.3 Compound split

$T_{MG1}$  and  $T_{MG2}$  show positive torque to propel the vehicle(Figure 5a).  $\omega_{MG2}$  increases with the increasing vehicle velocity and reaches its maximum speed at  $V=70\text{kph}$ (Figure 5b). Therefore, EV mode operation is possible below  $V=70\text{kph}$ . From Figure 5c, it is noted that  $P_{MG1}$  is negative and  $P_{MG2}$  is positive, which means that MG1 is used as the generator while MG2 is used as the motor. The power generated by the MG1 is transmitted to the MG2, which means that the power is circulated.

It is found from Figure 3 ~ Figure 5 that the EV mode can be realized simply by control of the MG2 for the input split system, but the MG1 and MG2 control is required for both the output split and the compound split system. In addition, the power circulation characteristics should be considered for the compound split HEV.

## 3.2 HEV mode

### 3.2.1 Input split

The motor MG1 shows the negative value,  $T_{MG1}=-33\text{Nm}$  meanwhile the engine torque remains at the optimal operation point,  $T_e=100\text{Nm}$ (Figure 6a).  $\omega_{MG1}$  decreases as the vehicle velocity increases (Figure 6b). The engine speed is controlled at the optimal operation point,  $\omega_e=200\text{rad/sec}$  by the MG1.

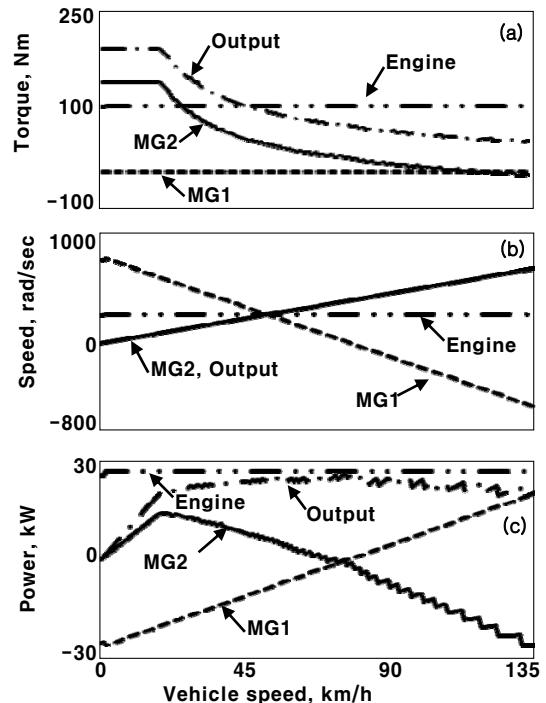


Figure 6 Input split HEV for HEV mode

It is seen from Figure 6c that the MG1 generates the power( $P_{MG1}<0$ ) and the MG2 is used as the motor( $P_{MG2}>0$ ) below  $V=80$ kph. It is noted that the input split HEV encounters, so called “mechanical point” at  $V=80$ kph where  $P_{MG1}$  and  $P_{MG2}$  become zero. The power that is generated from the MG1 is transmitted to the MG2 to propel the vehicle. However, if  $P_{MG1}>P_{MG2}$ , the extra power should be stored in the battery, which may be a problem when the battery state of charge(SOC) is above the upper limit. Above  $V=80$ kph, the role of MG1 and MG2 is changed, in other words, the MG1 is used as the motor( $P_{MG1}>0$ ) and vice versa.

### 3.2.2 Output split

The engine torque  $T_e$  increases from  $T_e=80$ Nm and reaches the target value  $T_e=100$ Nm at  $V=45$ kph(Figure 7a).  $T_{MG1}$  shows the positive torque to propel the vehicle until the vehicle velocity reaches  $V=100$ kph while  $T_{MG2}$  shows the negative torque for the generation. The engine speed  $\omega_e$  does not reach the target speed  $\omega_e=200$ rad/sec until  $V=45$ kph due to the torque limit of the MG1 and MG2.

It is seen from Figure 7c that the motor power  $P_{MG2}$  shows the negative value ( $P_{MG2}<0$ ) for the generation until  $V=100$ kph while the  $P_{MG1}$  shows the positive value to propel the vehicle. After the mechanical point( $V=100$ kph), the role of the MG1 and MG2 is changed showing that the MG2 plays as the motor and vice versa.

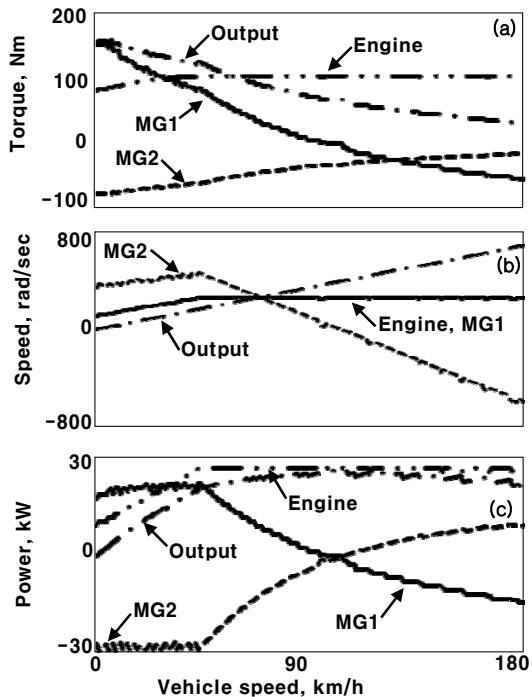


Figure 7 Output split HEV for HEV mode

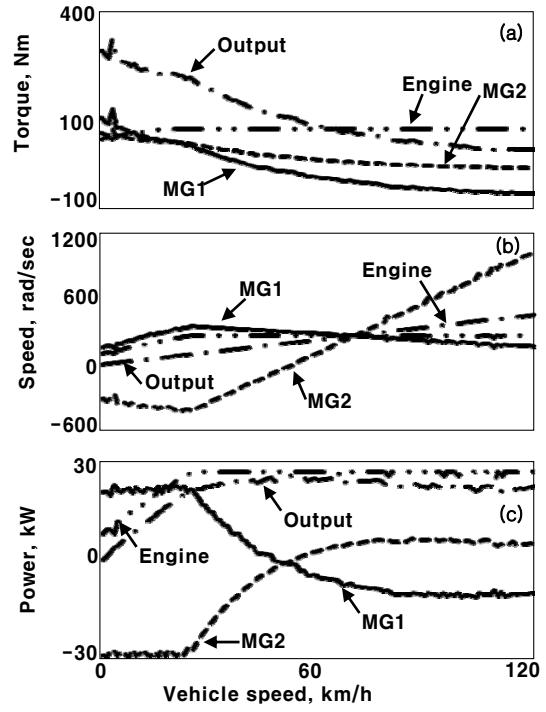


Figure 8 Compound split HEV for HEV mode

### 3.2.3 Compound split

The output torque,  $T_{MG1}$  and  $T_{MG2}$  decrease from the start as the vehicle velocity increases, for the compound split HEV(Figure 8a). The engine speed does not show the target speed at low vehicle speed due to the motor torque limitation, and reaches the target speed  $\omega_e=200$ rad/sec over  $V=30$ kph.  $\omega_{MG2}$  increases rapidly and reaches the maximum speed at  $V=120$ kph(Figure 8b). This means that the compound split configuration investigated in this study can only be used below  $V=120$ kph. It is seen from Figure 8c that the MG2 generates the power below  $V=55$ kph while the MG1 is used as the motor. After the mechanical point at  $V=55$ kph, the role of the MG1 and MG2 is reversed, and the MG2 is used as the motor to propel the vehicle, and vice versa.

In Figure 9, the MG1 and MG2 power, and the output power are compared for various power split configuration. It is seen from Figure 9a that the MG1 and MG2 power of the input split configuration is smaller than the others when vehicle starts( $V=0\sim 30$  kph) showing the output torque which is similar to those of the others (Figure 6a~ Figure 8a). And the mechanical point of the compound split can be obtained at relatively lower vehicle velocity compared with those of the input and output split structure. In addition, it is noted that smaller motor size can be used if the compound split structure is adopted for  $V=30\sim 90$ kph. The compound split structure can produce the highest output power among the three

configuration below  $V=90\text{kph}$ . However, the compound split structure cannot be used over  $V=120\text{kph}$ . Thus, the dual mode power split configuration which is expected to has each power split configuration's merits by combining those power split configurations can be used to improve overall system efficiency[5].

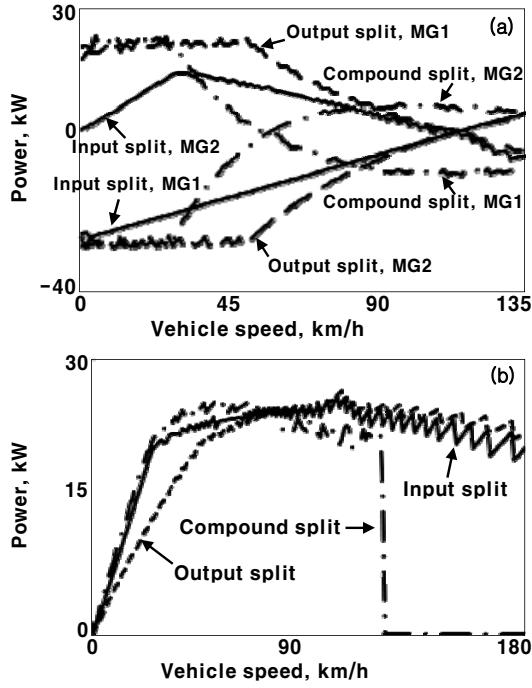


Figure 9 Electric machine and output power for HEV mode

### 3.3 Regenerative braking mode

#### 3.3.1 Input split

The regenerative braking is performed by the MG2 which is connected to the output shaft.  $T_{MG1}$  is zero since the MG1 does not work (Figure 10a).  $\omega_{MG2}$  increases with the output speed while the  $\omega_{MG1}$  decreases in the negative direction (Figure 10b). The regenerative power (Figure 10c) is generated by the MG2.

#### 3.3.2 Output split

$T_{MG1}$  shows the negative torque during the regenerative braking while  $T_{MG2}$  shows the positive torque by the lever relationship (Figure 11a).  $\omega_{MG2}$  decreases in the negative direction with the increasing output speed.  $\omega_{MG1}$  shows zero since the MG1 is connected to the engine, which remains off (Figure 11b).  $P_{MG1}$  remains zero (Figure 11c). This indicates that  $T_{MG1}$  should be controlled as the brake during the regenerative braking.

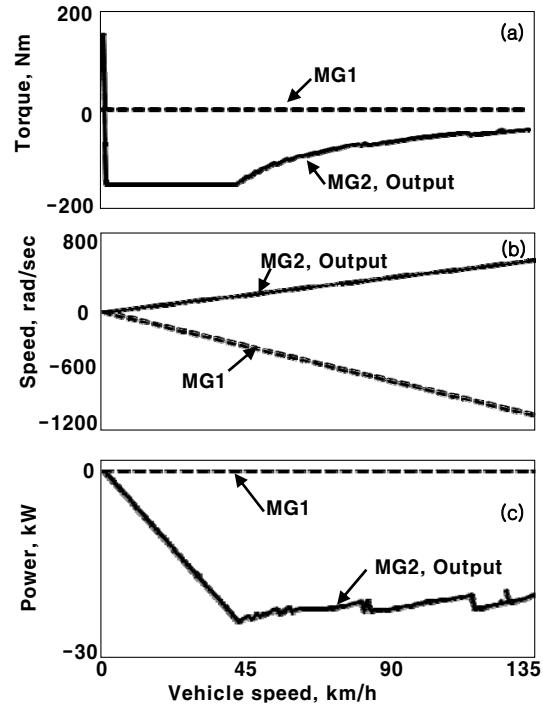


Figure 10 Input split HEV for REGEN mode

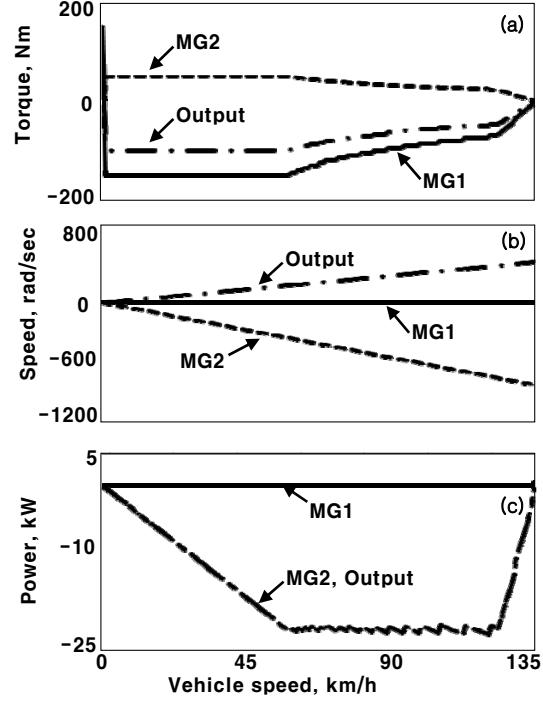


Figure 11 Output split HEV for REGEN mode

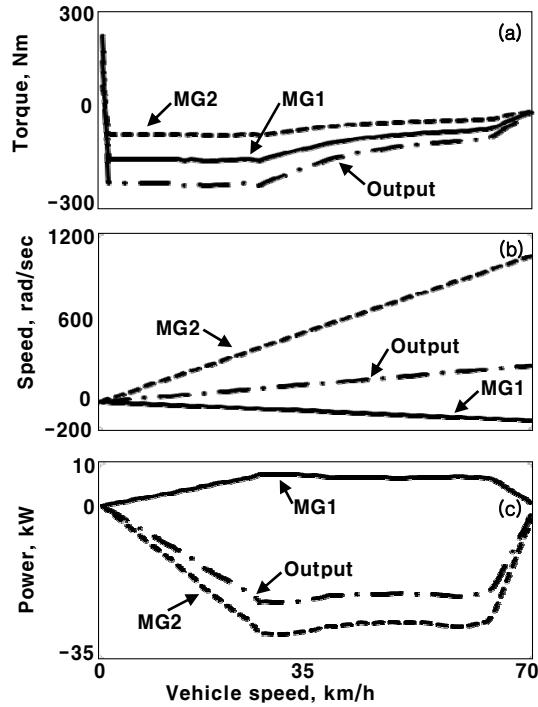


Figure 12 Compound split HEV for REGEN Mode

### 3.3.3 Compound split

$T_{MG1}$  and  $T_{MG2}$  show the negative torque during the regenerative braking (Figure 12a).  $\omega_{MG2}$  increases rapidly and reaches the maximum speed at  $V=70\text{kph}$ . Above  $V=70\text{kph}$ , the regenerative braking can not be implemented any more. It is found from Figure 12c that  $P_{MG1}$  shows the positive power, but  $P_{MG2}$  shows the negative power, which results in the power circulation.

From the simulation results in Figure 10 ~ Figure 12, it is seen that only the MG2 control is required for the input split configuration but both the MG1 and MG2 need to be controlled for the output and compound split configuration during the regenerative braking.

## 4 Conclusion

Power characteristics of the power split HEV are investigated using the lever analysis. For the analysis, generalized expression of the 4-node lever is derived to represent the torque and speed relationship. From the simulation results, it is found that the input split configuration has an advantage for the electric vehicle (EV) and regenerative braking (REGEN) mode. The compound split configuration has an advantage of using relatively small motors in the low to medium speed range, which gives the reduced

cost of power electronics. However, the compound split structure can not be used in the high speed range due to the maximum speed limit. It is expected that the simulation results of the power characteristics for the input, output and compound split configuration can be used in design of the power split type HEV with smaller motor size and lower fuel consumption while satisfying the demanded vehicle performance.

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