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Dual Kalman Filter for State of Charge Estimation of a Lead-acid Battery

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Abstract

A precise state of charge (SOC) estimation is necessary to improve the longevity, performance, reliability, density, and economics of batteries in conventional internal combustion engine vehicles and some electric vehicles. A Kalman filter is one of the techniques used to determine the SOC of the battery. For a nonlinear battery model, nonlinear Kaman filters such as an extended Kalman filter and a sigma point Kalman filter are used. However, the nonlinear Kalman filters that were used in other studies were very complicated to apply to the SOC estimation due to the complex nonlinear equations of the battery model. In this study, we represented a battery model with simple linear equations, which can represent the battery dynamics for a non-zero battery current. In this linear battery model, a model parameter was assumed to be varied with respect to the SOC. For this battery model, we applied a dual Kalman filter (DKF) method to estimate both the model parameter and the SOC. In the estimation of the battery model parameter, the internal resistance of the battery was estimated from the time constant characteristic of the battery terminal voltage at rest after discharging. Then the SOC was observed from the estimated internal resistance of the battery. As a result of that, we proved that the DKF can effectively estimate the SOC using the simple linear battery model.

Keywords: state of charge, lead-acid battery, dual Kalman filter

1 Introduction

Lead-acid batteries are still widely used in conventional internal combustion engine vehicles and some electric vehicles. In order to improve longevity, performance, reliability, density, and economy of batteries, the precise state of charge (SOC) estimation is required [1]. Several techniques have been studied for the determination of SOC, such as ampere-hour counting, measurement of the electrolytes physical properties, open circuit voltage, impedance spectroscopy, and Kalman filter [2].
The Kalman filter method can estimate SOC dynamically in real-time using a battery model [3]. For a nonlinear battery model, nonlinear Kalman filters have been applied such as an extended Kalman filter [1,4-8] and a sigma-point Kalman filter [9, 10].
However, the nonlinear battery models have complex nonlinear equations in SOC estimation

methods using the nonlinear Kalman filter [4-10]. For this reason, the SOC estimation algorithm cannot be easily implemented in practice.

In this study, a linear battery model was proposed to estimate the SOC by using a linear Kalman filter. In this battery model, a model parameter was assumed to be varied with respect to the SOC. This parameter of this battery model was also estimated by using another linear Kalman filter. This is the Dual Kalman filter (DKF) method to estimate both the model parameter and the SOC.

2 Battery Model

2.1 Battery model structure

A linear model structure was used for a discrete time lead-acid battery model. Equation (1) and (2) represent the linear battery model structure.

$$s_{k+1} = s_k - \left(\frac{\eta_i T}{C}\right) i_k + w_k \tag{1}$$

$$\begin{aligned} y_k &= \mathrm{OCV}(s_k) + Ri_k + v_k \\ &= K_1 s_k + K_0 + Ri_k + v_k \end{aligned} \tag{2}$$

Where s is the SOC state, i is the battery current, y is the battery terminal voltage, R is the battery internal resistance, and OCV(s_k), the open-circuit voltage as a function of SOC, can be computed linearly as

$$\mathrm{OCV}(s_k) = K_1 s_k + K_0 \tag{3}$$

In addition, η_i is the coulombic efficiency($\eta_i = 1$ for discharge, and $\eta_i \leq 1$ for charge), T is the sampling period, C is the nominal capacity, and w and v are independent, zero-mean, Gaussian noises for process and measurements, respectively.

2.2 Experiments

A lead-acid battery with a nominal voltage of 8 V and a nominal capacity of 100 Ah was tested. For this test, an electronic load was used at room temperature. This electronic load can consume the battery current with an accuracy of ±0.3%. The battery terminal voltage and current were measured by a DAQ system, using Labview® from National Instruments.

The battery was tested on a current profile. This current profile is shown in Figure 1. In the current profile, the battery was discharged on the constant current pulse and rest sequences. For the discharge, the battery was discharged from 100 down to 10 A. According to the discharge current profile, the battery terminal voltage was decreased and increased. This battery terminal voltage profile is shown in Figure 2. Furthermore, SOC profile of battery was obtained by Ah counting method. Figure 3 shows this SOC profile of battery based on the discharge current profile.

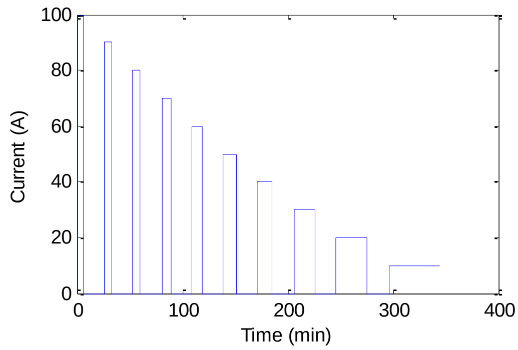

Figure 1: Discharge current profile

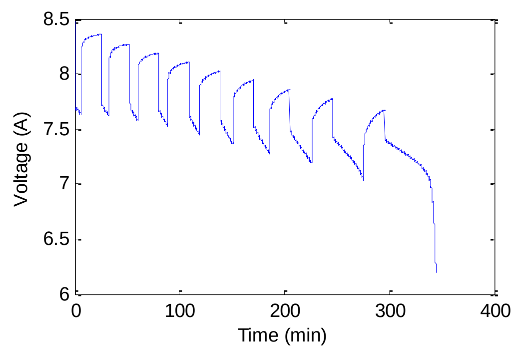

Figure 2: Discharge voltage

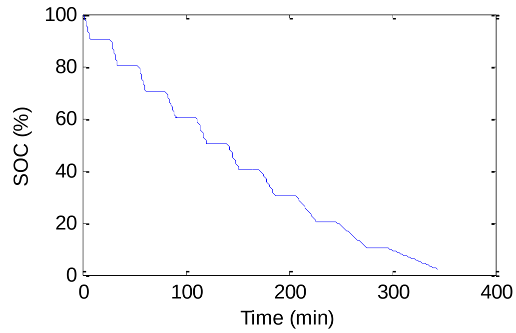

Figure 3: Discharge SOC

2.3 Model parameters identification

Model parameters were determined by applying the least-square method. Here, the only internal resistance of the battery was assumed to be varied with respect to the SOC. For using the least-square method, the battery model output equation can be represented as a regression model as shown in Eq. (4).

$$\begin{aligned} y_k &= \mathrm{OCV}(s_k) + Ri_k \\ &= K_1 s_k + K_0 + Ri_k \\ &= [s_k \quad 1 \quad i_k][K_1 \quad K_0 \quad R]^T \\ &= \varphi_k^T \theta \end{aligned} \tag{4}$$

For N number of observations, Eq. (4) can be written as

$$\mathbf{Y} = \mathbf{\Phi} \theta \tag{5}$$

where Y = $[y_1, y_2, \ldots, y_N]^T$ and $\Phi = [\varphi_1^T, \varphi_2^T, \ldots, \varphi_N^T]^T$. As a result, the parameters can be obtained from $\theta = (\Phi^T\Phi)^{-1}\Phi^T Y$ for a non-singular $(\Phi^T\Phi)$. The observations in which the current was not zero were relevant to this determination of parameters, because this linear battery model cannot represent the slow variation of the effect of the time constant when the battery current is zero. Moreover, the observations of 100 A discharge current, which is the first pulse in the discharge current profile, was only considered for identifying the initial internal resistance of the battery.

Table 1 shows the result of parameters identification. For these parameters, the modeling results are shown in Figure 4 and Figure 5. In Figure 4, the battery model represents the battery output voltages with respect to the 100 A discharge current. However, in Figure 5, this battery model cannot represent the accurate battery output voltages for other discharge current pulses. Thus, the battery internal resistance, R, should be changed according to the SOC. This internal resistance can be estimated from the time constant value of the battery output voltage at zero battery current. Indeed, the variations of the battery output voltage are different in each rest period after discharging current pulses as shown in Figure 2 and Figure 5.

In Figure 4, the initial time constant value of the battery output voltage in the rest period, τ_0, is shown in Table 1. An estimation method of this time constant value was represented at the next section.

Table 1: Battery model parameters

K_1	K_0	R_0	τ_0
0.7023	7.7647	-7.6572×10^{-3}	2294

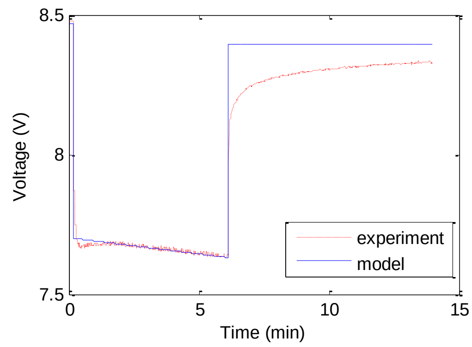

Figure 4: Modeling result in 100 A discharge current pulse

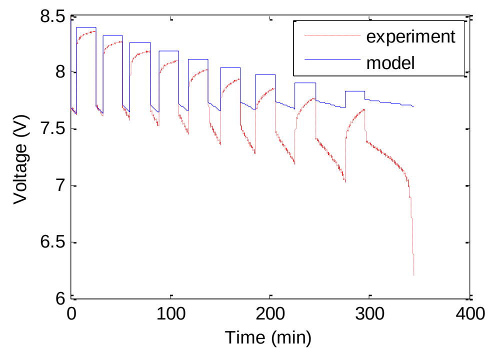

Figure 5: Modeling result in whole discharge current profile

3 Dual Kalman Filter

Kalman filters are widely used in estimation problems [11]. In this study, the SOC was estimated by using a DKF. Figure 6 shows the block diagram of the DKF used in this study. At first, the internal resistance of the battery was estimated from the battery voltage, current, and the previous SOC estimation. Then, from this estimated internal resistance, the SOC estimation can be updated by using a linear Kalman filter.

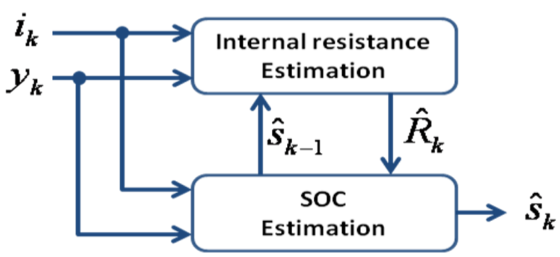

Figure 6: Block diagram of the dual Kalman filter

3.1 Internal resistance estimation

The internal resistance of the battery can be estimated from the time constant of battery output voltage based on the initial modeled internal resistance and time constant. Therefore, the internal resistance, R, can be represented as

$$R = \alpha R_0 \frac{\tau}{\tau_0} \tag{6}$$

where τ is the estimated time constant and α is the constant coefficient of the internal resistance variation.

3.1.1 Time constant estimation

In order to estimate the time constant value of the battery output voltage, the first order discrete time model in Eq. (7) was derived from the first order continuous time model in Eq. (8).

$$\begin{aligned} G(z) &= \frac{k\tau(1-e^{-T/\tau})}{z-e^{-T/\tau}} \\ &= \frac{b}{z-a} \end{aligned} \tag{7}$$

$$G(s) = \frac{k}{s+1/\tau} \tag{8}$$

Where T is the sampling period, τ is the time constant, k is the DC gain, $a = \exp(-T/\tau)$, and $b = k\tau(1-\exp(-T/\tau))$.

The discrete time model in Eq. (7) can be represented to the difference equations as

$$y_k = ay_{k-1} + bu_{k-1} \tag{9}$$

$$y_{k-1} = ay_{k-2} + bu_{k-2} \tag{10}$$

By subtracting Eq. (10) from Eq. (9) and assuming $u_{k-1} = u_{k-2}$, Eq. (11) can be obtained.

$$\begin{aligned} y_k - y_{k-1} &= a(y_{k-1} - y_{k-2}) + b(u_{k-1} - u_{k-2}) \\ &\approx a(y_{k-1} - y_{k-2}) \end{aligned} \tag{11}$$

From the time constant model in Eq. (11), the time constant τ can be estimated by using a Kalman filter. Eq. (12-13) represent the time update equation and Eq. (14-16) represent the measurement update equation in the Kalman filter [12, 13].

$$\hat{a}_k^- = \hat{a}_{k-1} \tag{12}$$

$$Cov_{\tilde{a},k}^- = Cov_{\tilde{a},k-1} + Cov_{a,w} \tag{13}$$

$$L_{a,k} = Cov_{\tilde{a},k}^- C_{a,k}^T [C_{a,k} Cov_{\tilde{a},k}^- C_{a,k}^T + Cov_{a,v}]^{-1} \tag{14}$$

$$\hat{a}_k^+ = \hat{a}_k^- + L_{a,k}[y_k - y_{k-1} - \hat{a}_k^- C_{a,k}] \tag{15}$$

$$Cov_{\tilde{a},k}^+ = (1 - L_{a,k} C_{a,k}) Cov_{\tilde{a},k} \tag{16}$$

where $C_{a,k} = y_{k-1} - y_{k-2}$, and $Cov_{\tilde{a}}$, $Cov_{a,w}$, and $Cov_{a,v}$ are the covariance of the error, the process noise, and the measurement noise, respectively. Figure 7 shows the estimation result of $\hat{a} = \exp(-T/\hat{\tau})$. Thus, the time constant value can be computed as $\hat{\tau} = -T/\ln(\hat{a})$. Figure 8 shows the estimation result of the time constant.

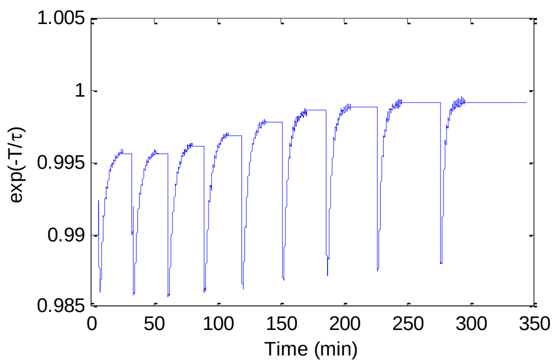

Figure 7: Estimation result of $\hat{a} = \exp(-T/\hat{\tau})$

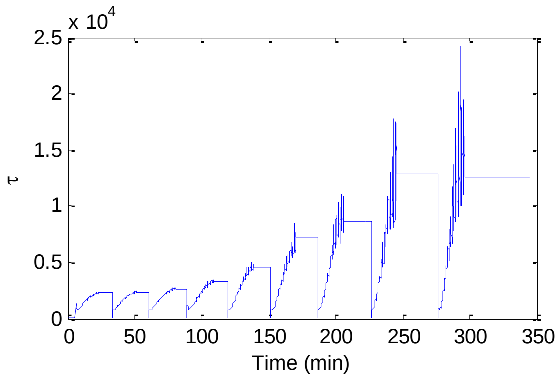

Figure 8: Estimation result of time constant

3.1.2 Internal resistance estimation

From the estimation result of the time constant, the internal resistance of the battery in Eq. (6) can be computed in Figure 9. Figure 10 represents the variation of the internal resistance of the battery with respect to the SOC.

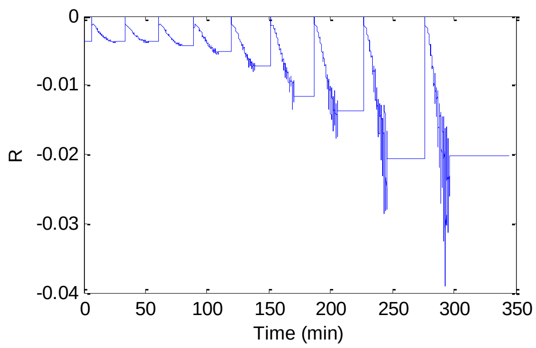

Figure 9: Estimation of the internal resistance

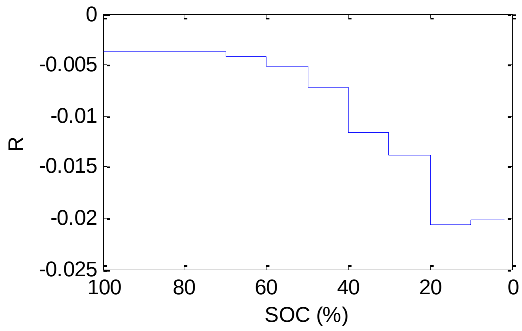

Figure 10: Estimation of the internal resistance with respect to the SOC

3.2 SOC Estimation

The SOC can be estimated from the estimated internal resistance of battery by applying a Kalman filter. Eq. (17-18) represent the time update equation and Eq. (19-21) represent the measurement update equation in the Kalman filter [12, 13].

$$\hat{s}_k^- = \hat{s}_{k-1} - \left(\frac{\eta_i T}{C}\right) i_{k-1} \qquad (17)$$

$$Cov_{\tilde{s},k}^- = Cov_{\tilde{s},k-1} + Cov_{s,w} \qquad (18)$$

$$L_{s,k} = Cov_{\tilde{s},k}^- C_{s,k}^T [C_{s,k} Cov_{\tilde{s},k}^- C_{s,k}^T + Cov_{s,v}]^{-1} \qquad (19)$$

$$\hat{s}_k^+ = \hat{s}_k^- + L_{s,k}[y_k - (\hat{s}_k^- C_{s,k} + K_0 + \hat{R} i_k)] \qquad (20)$$

$$Cov_{\tilde{s},k}^+ = (1 - L_{s,k} C_{s,k}) Cov_{\tilde{s},k} \qquad (21)$$

where $C_{s,k} = K_1$, and $Cov_{\tilde{s}}$, $Cov_{s,w}$, and $Cov_{s,v}$ are the covariance of the SOC error, the process noise, and the measurement noise, respectively.

From the experimental data, the linear battery model, and the DKF, the SOC of the battery was estimated. In this estimation, the initial values of the DKF are shown in Table 2 and 3. The battery output voltage and SOC estimation results are displayed in Fig. 11 and 12, respectively. As shown in Fig. 13, the SOC estimation errors quickly converge into the ± 3% error bound.

Table 2: Simulation condition of the $\hat{\tau}$ estimation

$\hat{a}_0 = \exp(-T/\hat{\tau})$	$Cov_{\tilde{a},0}$	$Cov_{a,w}$	$Cov_{a,v}$
0.99	1	1	0.001

Table 3 Simulation condition of the SOC estimation

$\hat{s}_0$	$Cov_{\tilde{s},0}$	$Cov_{s,w}$	$Cov_{s,v}$
50 %	1	1	1

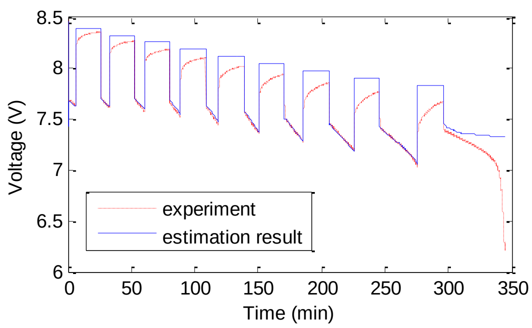

Figure 11: Battery voltage estimation result

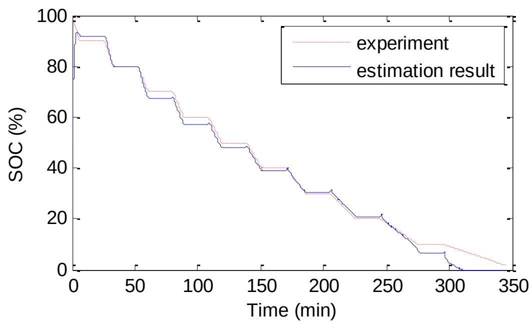

Figure 12: SOC estimation result

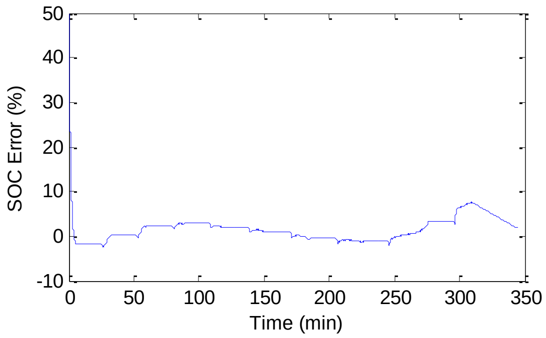

Figure 13: SOC estimation error

4 Conclusion

In this study, a lead-acid battery was modeled by a simple linear equation. This linear battery model represented the battery output voltage according to the internal resistance and the battery current input when the current was not equal to zero. For this linear battery model, the SOC of the battery was estimated by using the DKF. At first, the internal resistance of the battery was estimated from the time constant estimation by using the Kalman filter. Then the SOC was estimated using the internal resistance estimation by applying the Kalman filter. Consequently, the linear battery model and the DKF algorithm can effectively estimate the SOC.

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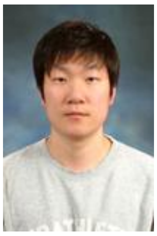

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