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Super cap battery simulation for real time applications based on model reduction using balanced truncation

Anton Tonchev[1], Mario Schweiger[2], Stefan Jakubek[3]
[1]*AVL Graz, Hans List Platz 1, 8020 Graz, Austria, anton.tonchev@avl.com*
[2]*AVL Graz, Hans List Platz 1, 8020 Graz, Austria, mario.schweiger@avl.com*
[3]*Vienna University of Technology, Wiedner Hauptstr. 8-10, 1040 Vienna, Austria, stefan.jakubek@tuwien.ac.at*

Abstract

Over the last 30 years, many industries dramatically improved their energy efficiency. Transportation is one exception. The global oil consumption shifted dramatically towards transportation (1971: ca. 1/3 of total oil demand; 2008: ca. 1/2). Light-duty vehicles (passenger cars, sport-utility vehicles, minivans and light trucks) account for more than 45 % of the transport sector's total oil consumption. With the emergence of hybrid vehicles and their market penetration worldwide, the end of rapid demand growth from this segment is in sight. The article sheds light on the chances and challenges of using energy storage system modeling and battery simulation for developing hybrid electric and pure electric vehicles. The task is to develop a physical model describing the complex energy storage systems. Describing analytically the exact dynamics of the system using expressions that are too cumbersome, too messy, or too ill-conditioned is not useful. In this paper a survey on balanced truncation model order reduction for modeling of complex energy storage systems such as super cap batteries is presented. Using this method the complex dynamic behaviour of a super cap battery with multiply degrees of freedom is transfered into a energy storage system model with exactly two degrees of freedom. The efficiency of this method is demonstrated by numerical experiments.

Keywords: super cap, battery simulation, balanced-truncate model oder reduction, hybrid electric vehicles

1 Introduction

A major concern of the engineers for the development of hybrid electric vehicles is the reproducability of the dynamic behavior of the energy storage system and the availability of such energy storage systems in the development process. The dynamic characteristic of the energy source, the dependency of the actual open and closed circuit voltage on the state of charge (SOC), on the current charge and discharge rate, on the internal battery temperature and the voltage recovery phenomenon upon interrupted charging/discharging have to be considered.

Electrochemical models are accurate but inherently suffer from the long simulation time required in practice. Consequently, more efficient battery models have been proposed in recent years. Macro-models for lithium-ion batteries are available, where the battery is modeled by a circuit comprising of voltage sources and linear passive elements. Since numerical complex computations of energy storage system are time consuming, the present paper proposes a state-space model of super cap battery, which is transformed into another state-space model with the help of balanced-truncated model order reduction method. The battery simulation model is solved numerically per every discrete time step. The computed output voltage of the battery model represents the demand voltage voltage for the power electectronics of the battery simulator.

2 Real time battery simulation

The battery simulator is used for validation and optimization of electric motors and full hybrid electric vehicle powertrains considering different battery behaviors. The battery simulator acts as DC-voltage supply and emulates the terminal voltage (U) of high performance energy storage systems in real time, e.g. super caps, Li-Ion batteries and fuel cells, see Fig. 1.

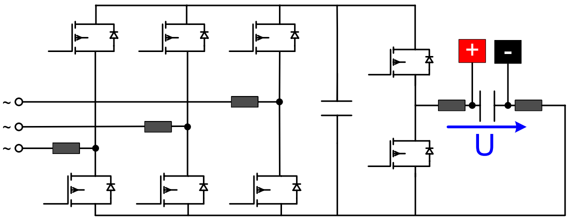

Figure 1: Real time battery simulation

Different battery models can be parameterized based on the test bed measurements and the dynamic behaviour of the energy storage systems can be reproduced with the battery simulator. To be able to simulate the behavior of complex super cap batteries a high quality and real-time model has to be used. To fullfil this requirements the battery simulation is optimzed via balanced and truncate model order reduction without quality losses regarding the simulated dynamics of the complexity of the energy storage system.

2.1 Advanced battery model

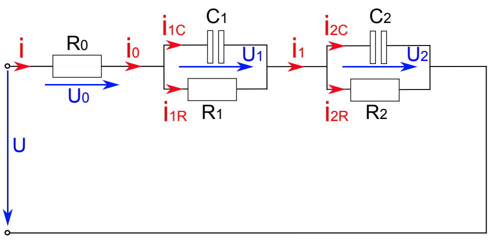

Figure 2: Advanced battery model

Applying the first current law of Kirchoff to the advaced battery model according to Fig. 2 one can get

$$
\begin{aligned}
i_0 &= i_{1R} + i_{1C} \\
i_1 &= i_{2R} + i_{2C},
\end{aligned}
\qquad (1)
$$

with the electrical current $i = i_0 = i_1$, the electrical currents i_{1R} and i_{2R} through the resistors R_1 respectively R_2 and the eltrical currents i_{1C} and i_{2C} through the capacitors C_1 respectively C_2. Additionally the following five equations can be written

$$
\begin{aligned}
&i_0 = \frac{U_0}{R_0}, \quad i_{1R} = \frac{U_1}{R_1}, \quad i_{2R} = \frac{U_2}{R_2}, \\
&i_{1C} = C_1\,\dot{U}_1, \quad i_{2C} = C_2\,\dot{U}_2.
\end{aligned}
\qquad (2)
$$

Considering (1) and (2) it follows

$$
\begin{aligned}
i &= \frac{U_1}{R_1} + C_1\,\dot{U}_1, \\
i &= \frac{U_2}{R_2} + C_2\,\dot{U}_2.
\end{aligned}
$$

Finally one can write the state space eqution of the system (Fig. 2)

$$\dot{x} = A\,x + B\,u \qquad (3)$$

as follows

$$
\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1\,C_1} & 0 \\ 0 & -\frac{1}{R_2\,C_2} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} \\ \frac{1}{C_2} \end{bmatrix} \begin{bmatrix} i \\ i \end{bmatrix} \qquad (4)
$$

with the state space vector $x = [U_1 \;\; U_2]^T$, the system matrix

$$
A = \begin{bmatrix} -\frac{1}{R_1\,C_1} & 0 \\ 0 & -\frac{1}{R_2\,C_2} \end{bmatrix}, \qquad (5)
$$

the input matrix $B = [\frac{1}{C_1} \;\; \frac{1}{C_2}]^T$ and the control vector $u = [i \;\; i]^T$.

The output equation of the system

$$y = C\,x + D\,u \qquad (6)$$

states as follows

$$
U = [1 \;\; 1] \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} + [R_0 \;\; 0] \begin{bmatrix} i \\ i \end{bmatrix} \qquad (7)
$$

with the output matrix $C = [1 \;\; 1]$ and the matrix $D = [R_0 \;\; 0]$.

2.2 Super cap battery model

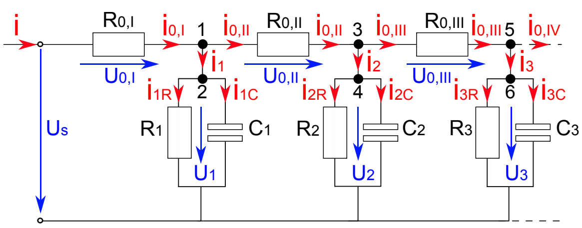

Figure 3: Super cap battery model

Applying the second current law of Kirchoff for the point 1 and 2 of the super cap battery model according to Fig. 3 one get

$$i_{0,I} = i_{0,II} + i_1$$
$$i_1 = i_{1R} + i_{1C}$$

or

$$i_{0,I} = i_{0,II} + i_{1R} + i_{1C}, \qquad (8)$$

with the electrical current i_{1R} through the resistor R_1 and the elctrical current i_{1C} through the capacitor C_1. Additionally the following four equations can be written

$$i_{0,I} = \frac{U_{0,I}}{R_{0,I}}, \qquad i_{0,II} = \frac{U_{0,II}}{R_{0,II}},$$
$$i_{1R} = \frac{U_1}{R_1}, \qquad i_{1C} = C_1 \dot{U}_1.$$

Considering the last four equations one can get with the help of (8)

$$i = \frac{U_{0,II}}{R_{0,II}} + \frac{U_1}{R_1} + C_1 \dot{U}_1,$$

with $i = i_{0,I}$ and $U_{0,II} = U_1 - U_2$. Finally it can be written

$$\dot{U}_1 = -\left(\frac{1}{R_{0,II}\, C_1} + \frac{1}{R_1\, C_1}\right) U_1 + \frac{1}{R_{0,II}\, C_1} U_2. \qquad (9)$$

Applying the second current law of Kirchoff for the point 3 and 4 of the super cap battery model according to Fig. 3 one get

$$i_{0,II} = i_{0,III} + i_2$$
$$i_2 = i_{2R} + i_{2C}$$

or

$$i_{0,II} = i_{0,III} + i_{2R} + i_{2C}, \qquad (10)$$

with the electrical current i_{2R} through the resistor R_2 and the elctrical current i_{2C} through the capacitor C_2. Additionally the following four equations can be written

$$i_{0,II} = \frac{U_{0,II}}{R_{0,II}}, \qquad i_{0,III} = \frac{U_{0,III}}{R_{0,III}},$$
$$i_{2R} = \frac{U_2}{R_2}, \qquad i_{2C} = C_2 \dot{U}_2.$$

Considering the last four equations one can get with the help of (10)

$$\dot{U}_2 = \frac{1}{R_{0,II}\, C_2} U_1 - \frac{1}{C_2}\left(\frac{1}{R_2} + \frac{1}{R_{0,II}} + \frac{1}{R_{0,III}}\right) U_2 + \frac{1}{R_{0,III}\, C_2} U_3, \qquad (11)$$

with $U_{0,II} = U_1 - U_2$ and $U_{0,III} = U_2 - U_3$.

If the super cap battery model containts of ten battery cells (Fig. 4) one gets for the voltage derivative $\dot{U}_{10}$ as follows

$$\dot{U}_{10} = \frac{1}{R_{0,X}\, C_{10}} U_9 - \frac{1}{C_{10}}\left(\frac{1}{R_{0,X}} + \frac{1}{R_{10}}\right) U_{10}. \qquad (12)$$

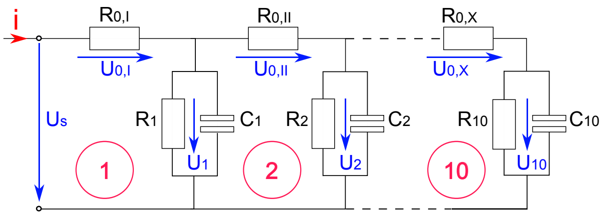

Figure 4: Super cap battery model of ten battery cells

Finally one can write the state space eqution of the modeled super cap battery consisting of ten cells (Fig. 4)

$$\dot{x}_s = A_s\, x_s + B_s\, u_s \qquad (13)$$

as follows

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \\ \vdots \\ \dot{U}_{10} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,10} \\ a_{2,1} & a_{2,2} & \dots & a_{2,10} \\ \vdots & \vdots & \ddots & \vdots \\ a_{10,1} & a_{10,2} & \dots & a_{10,10} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_{10} \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} [i(t)] \qquad (14)$$

with the state space vector $x_s = [U_1 \;\; U_2 \;\; \dots \;\; U_{10}]^T$, the system matrix

$$A_s = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,10} \\ a_{2,1} & a_{2,2} & \dots & a_{2,10} \\ \vdots & \vdots & \ddots & \vdots \\ a_{10,1} & a_{10,2} & \dots & a_{10,10} \end{bmatrix}, \qquad (15)$$

the input matrix $B_s = [\frac{1}{C_1} \;\; 0 \;\; \dots \;\; 0]^T$ and the load current $u_s = [i(t)]$.

The output equation of the system

$$y_s = C_s\, x_s + D_s\, u_s \qquad (16)$$

states as follows

$$U_s = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_{10} \end{bmatrix} + R_{0,\,I}\, i(t) \tag{17}$$

with the output matrix $C_s = [1 \;\; 0 \;\; \dots \;\; 0]$ and the matrix $D_s = [R_{0,\,I}]$.

3 Balanced realization and model reduction

For real time battery simulation of super caps consisting of ten battery cells the linear state space equations (14) and (6) are transformed into another form. The new coordinates are the so-called balanced coordinates x which can be computed by the followimg equation

$$x = T^{-1}\, x_s \tag{18}$$

with the transformation matrix T. The vector x represents the new coordinates of the balanced system, see Fig. 2. The balanced state space realization can be computed as follows

$$\begin{aligned} \tilde{A}_s &= T^{-1}\, A_s\, T, \\ \tilde{B}_s &= T^{-1}\, B_s, \\ \tilde{C}_s &= C_s\, T, \\ \tilde{D}_s &= D. \end{aligned} \tag{19}$$

For the computation of T the computing of the Controllability and Observability Gramians W_c respectively W_o are needed [1, 3, 4, 5, 6]. For a balanced state space systems the Controllability and Observability Gramians are diagonal. The Gramians satisfy the Lyaponov equations

$$\begin{aligned} A_s\, W_c + W_c\, A_s^T + B_s\, B_s^T &= 0 \\ A_s^T\, W_o + W_o\, A_s + C_s^T\, C_s &= 0 \end{aligned} \tag{20}$$

Once the linear state space equations of the super cap energy storage system are balanced the "truncation" of the balanced state components follows. In that case state components are deleted for which the corresponding diagonal entry of the Gramin is small. Finally the reduced state-space realization of the reduced model order is given by $\tilde{A}_s^{red}$, $\tilde{B}_s^{red}$, $\tilde{C}_s^{red}$ and D.

4 Simulation results

In this section some numerical computations are presented to illustrate the effectiveness and the quality of the model order reduction methods for super cap batteries. For that reason the load current $i = I$ is stepped from 0 to 100 A at time $t = 1$ s. In that case the output terminal voltages U_s and U of the original battery super cap model consisting of ten super cap battery cells respectively of the reduced order model have the same characteristics, see the first two plots of Fig. 5.

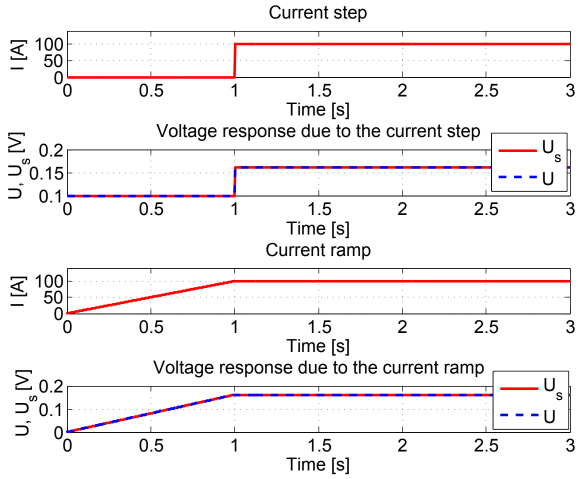

Figure 5: Step and ramp terminal voltage response (U_s) and U for the original super cap model respectively for the reduced order model for nominal load current of 100 A and ramp time of 1 s

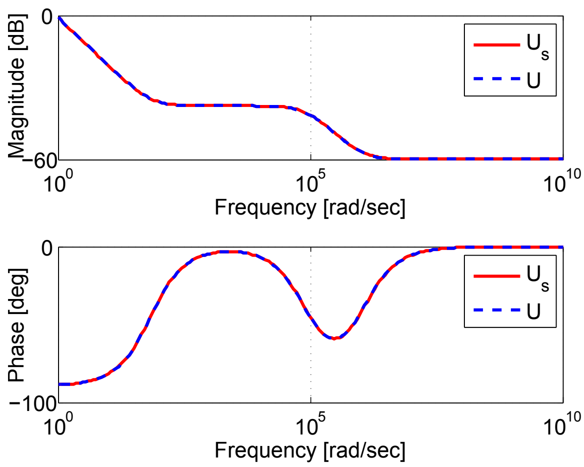

Figure 6: Bode diagram of the original and the reduced order model of the super cap battery

Both terminal voltages U_s and U coincidence in case that the load current i is ramped from 0 to 100 A too. The ramp time is 1 s, see the last two plots of Fig. 5. The quality of the simulation results is confirmed by comparing of the bode diagram of the original battery supper cap model (U_s) and the reduced oder model (U) too, see Fig. 6.

5 Conclusion

Modeling of complex physical and technical processes such as batteries, can lead to systems of very large model order, while the number of inputs and number of outputs could be typically small compared to the model order. Despite the ever increasing computational speed, simulation, optimization or real time controller design for such large-scale systems is dicult because of storage requirements and expensive computations.

In this paper a balanced truncation model order reduction method for modeling of linear time-invariant super cap batteries is presented. The approach is related to the controllability and observability Gramians that can be computed by solving the Lyapunov equation.

The simulation of the dynamic behavior of complex super cap batteries with a high quality should be also guaranteed. To fullfil this requirements the battery simulation is optimzed via balanced and truncate model order reduction without quality losses regarding the simulated dynamics of the complexity of the energy storage system.

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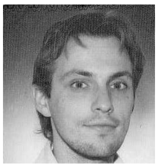
Authors

Dr. Anton Tonchev is Head of development hybrid and battery testing at AVL Graz, Austria. Research topics: hybrid and vehicle simulation, contact mechanics, friction, battery simulation and battery testing, finite element analysis.
Adress: AVL Graz, Hans List Platz 1, 8020 Graz, Austria. E-Mail: anton.tonchev@avl.com



Dr. Mario Schweiger, PhD in applied electrochemistry focused on material design for Lithium-Ion-Batteries. Head of application segment battery testing and battery simulation solutions.
Adress: AVL Graz, Hans List Platz 1, 8020 Graz, Austria. E-mail: mario.schweiger@avl.com

o. Univ. Prof. Dr. techn. Stefan Jakubek is full professor of control theory at the Institute for Mechanics and Mechatronics, Division of Control and Process Automation, Vienna University of Technoology. Research topics: sensor-fault-detection, system identification, battery simulation, non-linear predictive control.
Adress: Vienna University of Technology, Wiedner Hauptstr. 8-10, 1040 Vienna, Austria. E-mail: stefan.jakubek@tuwien.ac.at